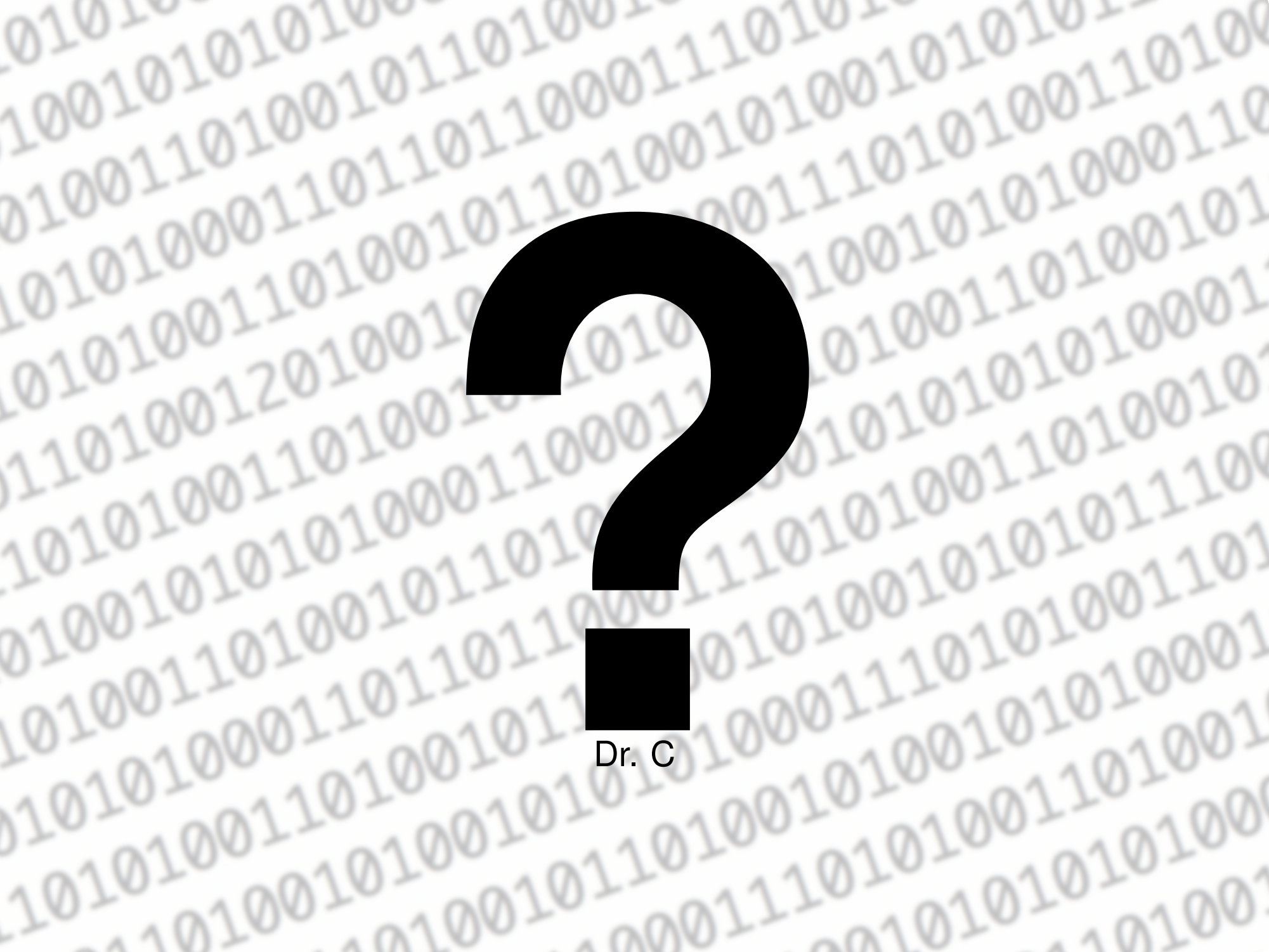

Dr. C

# I'm thinking of a number* from 0 to 7.

You can ask me yes-or-no questions to find it.

* A *whole* number. No $\pi$ here, MathPathogens!

# I'm thinking of a number* from 0 to 999,999.

You can ask me yes-or-no questions to find it.

* A *whole* number. No $\pi$ here, MathPathogens!

**Responder:** Secretly chooses a whole number:

$$0, 1, 2, \ldots, m - 1$$

**Questioner:** Asks yes-or-no questions.

**Responder:** Secretly chooses a whole number:

$$0, 1, 2, \ldots, m - 1$$

**Questioner:** Asks yes-or-no questions.

Can always phrase the questions as:

"Is your number in {list of numbers}?"

**Examples:**

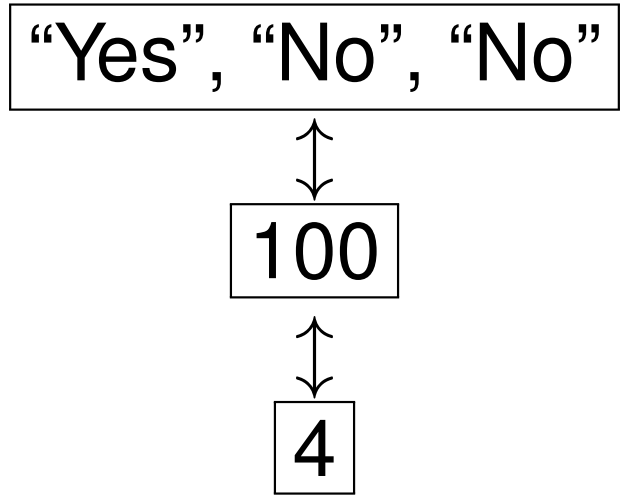
# A sample game

**Responder:** Chooses 4 (out of 0–7)

Start: $\{0, 1, 2, 3, 4, 5, 6, 7\}$

**interactive** / **on-line** **bisection**

“Yes” $\leftrightarrow$ 1, “No” $\leftrightarrow$ 0




We’ll determine the number from the answer vector.

So, each number must have a *different* vector.

What if you must ask all questions *before* hearing any answers? **(non-interactive** / **offline)**

What if you must ask all questions *before* hearing any answers? **(non-interactive / offline)**

Choose a number in $\{0, 1, 2, 3, 4, 5, 6, 7\}$

**Remember** your answers, in order:

What if you must ask all questions *before* hearing any answers? **(non-interactive / offline)**

Choose a number in $\{0, 1, 2, 3, 4, 5, 6, 7\}$

**Remember** your answers, in order:

1. Is your number in $\{\mathbf{4}, \mathbf{5}, \mathbf{6}, \mathbf{7}\}$?
2. Is your number in $\{\mathbf{2}, \mathbf{3}, \mathbf{6}, \mathbf{7}\}$?
3. Is your number in $\{\mathbf{1}, \mathbf{3}, \mathbf{5}, \mathbf{7}\}$?

What if you must ask all questions *before* hearing any answers? **(non-interactive / offline)**

Choose a number in $\{0, 1, 2, 3, 4, 5, 6, 7\}$

**Remember** your answers, in order:

1. Is your number in $\{\mathbf{4}, \mathbf{5}, \mathbf{6}, \mathbf{7}\}$?
2. Is your number in $\{\mathbf{2}, \mathbf{3}, \mathbf{6}, \mathbf{7}\}$?
3. Is your number in $\{\mathbf{1}, \mathbf{3}, \mathbf{5}, \mathbf{7}\}$?

Now let me guess. . .

How did I do this trick? By changing my viewpoint.

**Viewpoint #1:** More clever bisection

How did I do this trick? By changing my viewpoint.

**Viewpoint #1:** More clever bisection

**Responder:** Chooses 4 (out of 0 – 7)

**Start:** $\{0, 1, 2, 3, 4, 5, 6, 7\}$

**Q:** "Is your number in $\{4, 5, 6, 7\}$?"

$\{0, 1, 2, 3, 4, 5, 6, 7\}$

**Q:** "Is your number in $\{2, 3, 6, 7\}$?"

$\{0, 1, 2, 3, 4, 5, 6, 7\}$

**Q:** "Is your number in $\{1, 3, 5, 7\}$?"

$\{0, 1, 2, 3, 4, 5, 6, 7\}$

**Viewpoint #2:** Pick the answers first

**Responder:** Chooses 4 (out of 0 – 7)

| | | | |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 |
| 2 | 0 | 1 | 0 |
| 3 | 0 | 1 | 1 |
| 4 | 1 | 0 | 0 |
| 5 | 1 | 0 | 1 |
| 6 | 1 | 1 | 0 |
| 7 | 1 | 1 | 1 |

**Viewpoint #2:** Pick the answers first

**Responder:** Chooses 4 (out of 0 – 7)

| | | | |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 |
| 2 | 0 | 1 | 0 |
| 3 | 0 | 1 | 1 |
| 4 | 1 | 0 | 0 |
| 5 | 1 | 0 | 1 |
| 6 | 1 | 1 | 0 |
| 7 | 1 | 1 | 1 |

**Q:** "Is your number in $\{4, 5, 6, 7\}$?"

Yes! 1 _ _

**Viewpoint #2:** Pick the answers first

**Responder:** Chooses 4 (out of 0 – 7)

| | | | |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 |
| 2 | 0 | 1 | 0 |
| 3 | 0 | 1 | 1 |
| 4 | 1 | 0 | 0 |
| 5 | 1 | 0 | 1 |
| 6 | 1 | 1 | 0 |
| 7 | 1 | 1 | 1 |

**Q:** "Is your number in $\{2, 3, 6, 7\}$?"

No! 1 0 _

**Viewpoint #2:** Pick the answers first

**Responder:** Chooses 4 (out of 0 – 7)

| | | | |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 |
| 2 | 0 | 1 | 0 |
| 3 | 0 | 1 | 1 |
| 4 | 1 | 0 | 0 |
| 5 | 1 | 0 | 1 |
| 6 | 1 | 1 | 0 |
| 7 | 1 | 1 | 1 |

**Q:** "Is your number in $\{1, 3, 5, 7\}$?"

No! 1 0 0

**Viewpoint #2:** Pick the answers first

**Responder:** Chooses 4 (out of 0 – 7)

| | | | |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 |
| 2 | 0 | 1 | 0 |
| 3 | 0 | 1 | 1 |
| 4 | 1 | 0 | 0 |
| 5 | 1 | 0 | 1 |
| 6 | 1 | 1 | 0 |
| 7 | 1 | 1 | 1 |

It doesn't matter *which* vectors go with each number

| | | | |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 |
| 2 | 0 | 1 | 0 |
| 3 | 0 | 1 | 1 |
| 4 | 1 | 0 | 0 |
| 5 | 1 | 0 | 1 |
| 6 | 1 | 1 | 0 |
| 7 | 1 | 1 | 1 |

It doesn't matter *which* vectors go with each number

| | | | |
|---|---|---|---|
| 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 2 | 0 | 1 | 1 |
| 3 | 0 | 0 | 0 |
| 4 | 1 | 0 | 0 |
| 5 | 1 | 1 | 0 |
| 6 | 0 | 0 | 1 |
| 7 | 0 | 1 | 0 |

It doesn't matter *which* vectors go with each number
*as long as they are* ***distinct***

| | | | |
|---|---|---|---|
| 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 |
| 2 | 0 | 0 | 1 |
| 3 | 0 | 0 | 1 |
| 4 | 1 | 0 | 0 |
| 5 | 1 | 0 | 0 |
| 6 | 1 | 0 | 1 |
| 7 | 1 | 0 | 1 |

It doesn't matter *which* vectors go with each number
*as long as they are **distinct***

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |

# **Big** questions:

What is the minimum number of questions *required*, no matter what number I choose?

Is online or offline better?

What if I
**Lie?**

# What if Responder *lies?*

No, No, No $\longrightarrow$ No, No, Yes

000 $\longrightarrow$ 001

| | | | |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 |
| 2 | 0 | 1 | 0 |
| 3 | 0 | 1 | 1 |
| 4 | 1 | 0 | 0 |
| 5 | 1 | 0 | 1 |
| 6 | 1 | 1 | 0 |
| 7 | 1 | 1 | 1 |

How can we handle **one** lie?

| 0 | 0 | 0 | 0 |
|---|---|---|---|
| 1 | 0 | 0 | 1 |

The Responder said:

**001**

How can we handle **one** lie?

| | | | |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 |

The Responder said:

**001**

How can we handle **one** lie?

| 0 | 0 | 0 | 0 |
|---|---|---|---|
| 1 | 1 | 1 | 1 |

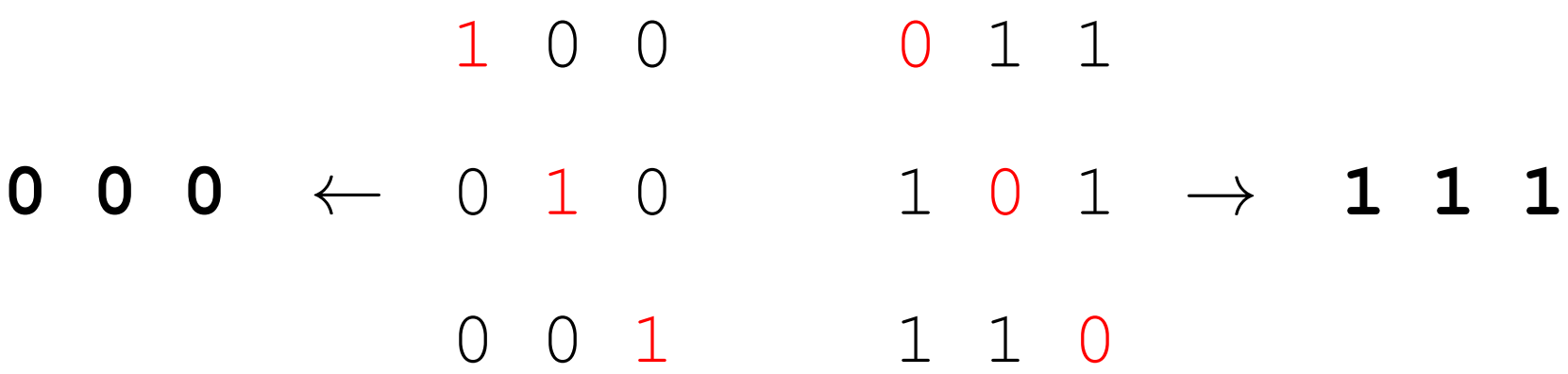
The Responder said:

**001**

To fix **one** mistake:

Every pair of response vectors must differ in at least 3 places.

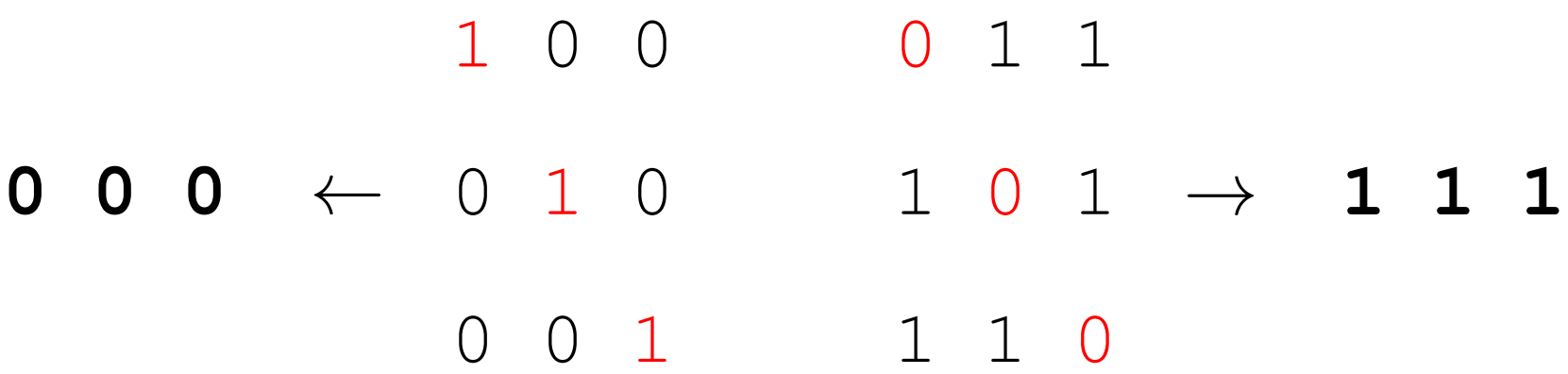
"Fix" by choosing the response vector *closest* to the Responder's answers.



To fix **one** mistake:

Every pair of response vectors must have **Hamming distance** at least 3.

"Fix" by choosing the response vector *closest* to the Responder's answers.



Repetition works for more than 2 options.

"Fix" by choosing the response vector *closest* to the Responder's answers:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 |

“Fix” by choosing the response vector *closest* to the Responder’s answers:

| | | | | | |
|---|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 |
| 2 | 0 | 0 | 1 | 1 | 1 |
| 3 | 0 | 0 | 0 | 0 | 0 |
| | 1 | 0 | 1 | 1 | 1 |

“Fix” by choosing the response vector *closest* to the Responder’s answers:

| | | | | | |
|---|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 |
| 2 | 0 | 0 | 1 | 1 | 1 |
| 3 | 0 | 0 | 0 | 0 | 0 |
| | 0 | 0 | 1 | 1 | 0 |

“Fix” by choosing the response vector *closest* to the Responder’s answers:

| | | | | | |
|---|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 |
| 2 | 0 | 0 | 1 | 1 | 1 |
| 3 | 0 | 0 | 0 | 0 | 0 |
| | 1 | 0 | 0 | 0 | 0 |

“Fix” by choosing the response vector *closest* to the Responder’s answers:

| | | | | | |
|---|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 |
| 2 | 0 | 0 | 1 | 1 | 1 |
| 3 | 0 | 0 | 0 | 0 | 0 |
| | 1 | 0 | 1 | 0 | 1 |

Pick a number 0 – 3:

| | | | | | |
|---|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 |
| 2 | 0 | 0 | 1 | 1 | 1 |
| 3 | 0 | 0 | 0 | 0 | 0 |

Pick a number 0 – 3:

| | | | | | |
|---|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 |
| 2 | 0 | 0 | 1 | 1 | 1 |
| 3 | 0 | 0 | 0 | 0 | 0 |

Is your number in $\{0, 1\}$?

Pick a number 0 – 3:

| | | | | | |
|---|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 |
| 2 | 0 | 0 | 1 | 1 | 1 |
| 3 | 0 | 0 | 0 | 0 | 0 |

Is your number in $\{0, 1\}$?

Pick a number 0 – 3:

| | | | | | |
|---|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 |
| 2 | 0 | 0 | 1 | 1 | 1 |
| 3 | 0 | 0 | 0 | 0 | 0 |

Is your number in $\{0, 2\}$?

Pick a number 0 – 3:

| | | | | | |
|---|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 |
| 2 | 0 | 0 | 1 | 1 | 1 |
| 3 | 0 | 0 | 0 | 0 | 0 |

Is your number in $\{0, 2\}$?

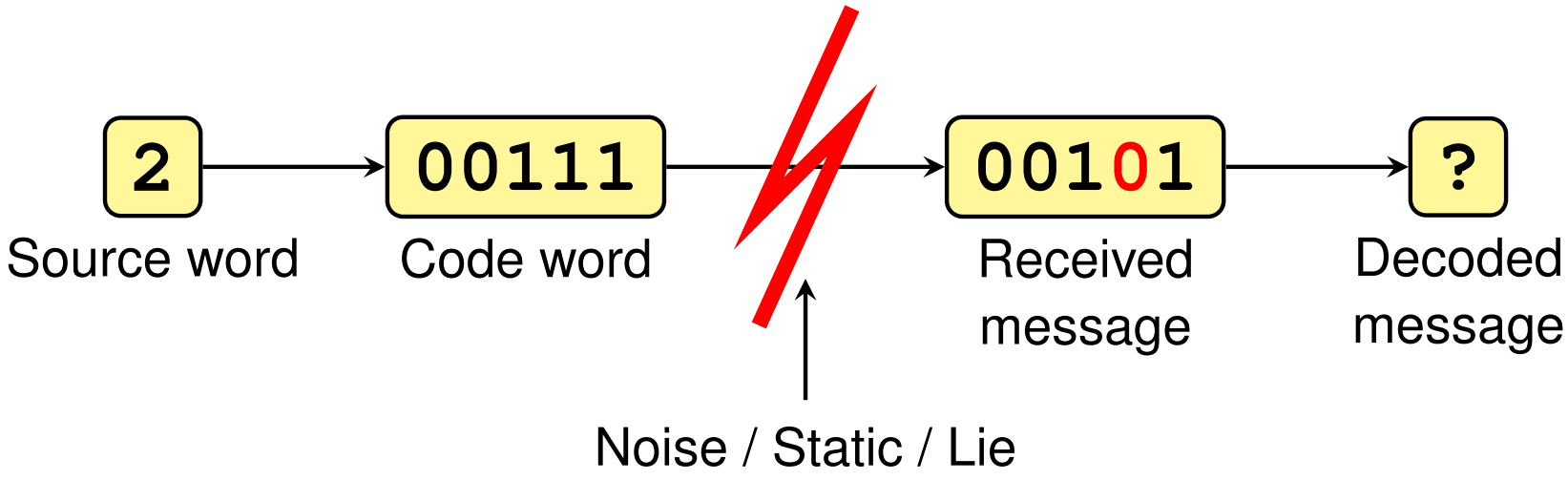
Pick a number 0 – 3:

| | | | | | |
|---|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 |
| 2 | 0 | 0 | 1 | 1 | 1 |
| 3 | 0 | 0 | 0 | 0 | 0 |

Is your number in $\{1, 2\}$?

# An **error-correcting code** with **minimum distance 3**:

| | | | | | |
|---|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 |
| 2 | 0 | 0 | 1 | 1 | 1 |
| 3 | 0 | 0 | 0 | 0 | 0 |



**Error-correcting codes** are used in:

- *Transmissions*: Wifi, USB, digital TV, cell phones, satellites, GPS
- *Storage*: Flash drives, hard disks, cloud storage
- *Computations*: Quantum computers
- Guessing games!

**We know the "best" games that can. . .**

- . . . fix 1 mistake (online *or* offline)
- . . . fix 2 or 3 mistakes (online only!)

**Things we don't know:**

- "Best" *anything* with 4 or more lies.

**Other interesting variations:**

- Questions have costs – how cheap can you be?
- Allow more than just "yes" or "no".
- Guess an interval of *real numbers*.

# Interested?

Take Guessing Games in Week 3!

# Questions?

$\{5, 6, 7, 8\}$?  $\{2, 3, 6, 7\}$?  $\{1, 3, 5, 7\}$?

# Pick a number 0 – 15:

{8, 9, 10, 11, 12, 13, 14, 15 }?

{4, 5, 6, 7, 12, 13, 14, 15 }?

{2, 3, 6, 7, 10, 11, 14, 15}?

{1, 3, 5, 7, 9, 11, 13, 15 }?

{1, 2, 4, 7, 9, 10, 12, 15 }?

{1, 2, 5, 6, 8, 11, 12, 15 }?

{1, 3, 4, 6, 8, 10, 13, 15 }?