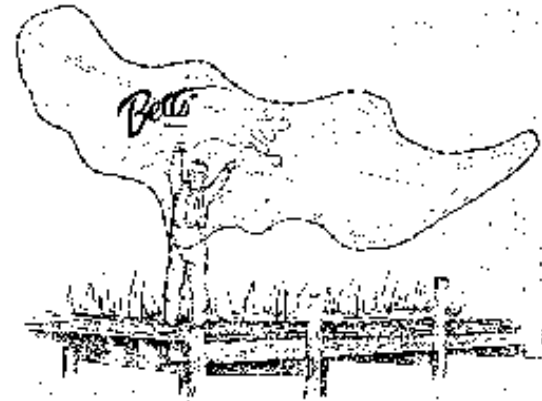


# Stopping Rule Selection (SRS) Theory Applied to Deferred Decision Making



**Mario Fifić**

*Grand Valley State University*

**Marcus Buckmann**

*Max Planck Institute for Human Development,  
Center for Adaptive Behavior and Cognition, Berlin*



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# Research Group



Angele Yazbec



Krysta Rydecki



Jeremy Winget



Kelly Dillon



Tessa Johnson



Kylie Loudenslager

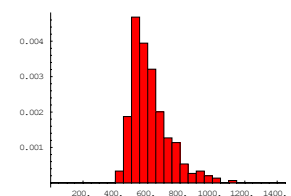
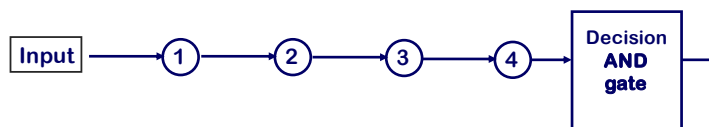


Kyle Zimmer

Andrew Humphrey

# Phenomena

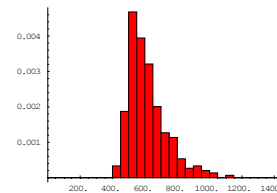
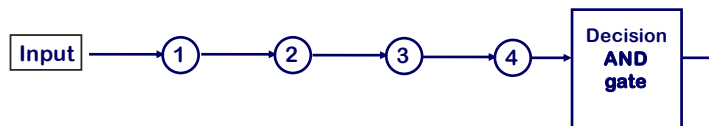
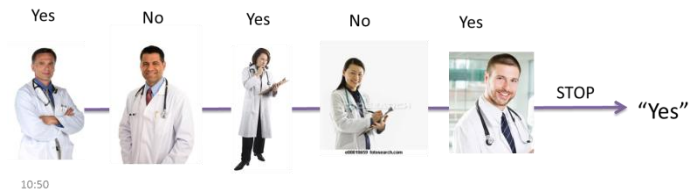
- Decide whether to buy a certain product or not, based on an optional number of reviews offered a service of web-online shopping providers such as Amazon or Buy.com.
- “ The Three Strikes and You-are-out” law(s) is advocated as punitive and deterrence strategy for reducing violent crime. The law is based on “three strikes” rule: if a newly convicted criminal had a record of two prior felony convictions, which is considered as a serious crime, the judge has to impose the maximum sentence for the third crime.
- How many doctor’s opinion?
- Buridan’s anecdote a donkey cannot decide between two equally attractive piles of hay, and starves to death
- Memory retrieval task



# Common denominator?

# Common denominator is a Stopping Rule

- Stopping Rule: a decision rule used to decide when to stop with evidence collection and for making final decisions.



# Deferred decision making task

Please take your buying decision  
on this product:



BUY



REVIEW



DON'T BUY

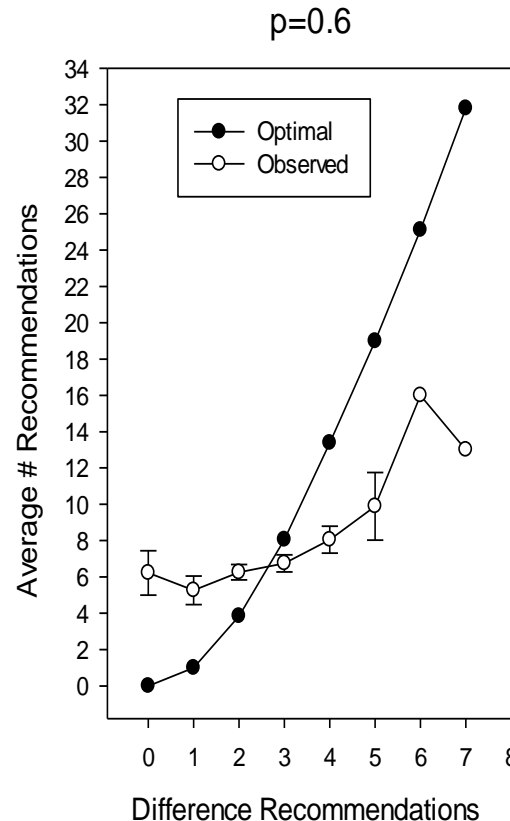
- In a deferred decision making task, subjects must decide whether or not to buy a product of unknown quality, basing their decision on reviews selected.
- The reliability of the reviews varied (for example .6, .75, .9).

# Stopping rule in judgment and decision making

- ***Optimal Rule belongs to Optimal Decision Maker:***
  - Stop when the expected value of loss is equal to, or lower than the expected loss associated with deferring the decision and collecting more evidence.
    - Requirements: calculate the so-called posterior odds in favor of the hypothesis that the option 1 is beneficial given the evidence (opinions) acquired from  $n$  sources  $\Omega \left( \frac{\text{Option 1}}{\text{Option 2}} \mid \text{opinion} \right)$ . The posterior odds would indicate the best decision for the finite number of opinions, if the costs and payoffs associated with the risky decision and the expected diagnostic value of a single opinion are considered.
    - the exact statistical representation of the environment nor optimization **is required**
- ***Non-optimal Rules belong to Suboptimal Decision Maker:***
  - *Critical Difference*: stop when a total sum of bipolar evidence reaches a critical value of ( $d$ ).
  - *Fixed Sample Size*: stop a collection on a certain number of evidence ( $s$ )
  - *Runs (Streak)*: stop on a repeated sequence of evidence of a certain size ( $r$ ).
    - Requirements: the exact statistical representation of the environment nor optimization **is not required**

# Are people optimal decision makers?

- (1) People bought too much or too little evidence (recommendations) (Pitz, 1968)



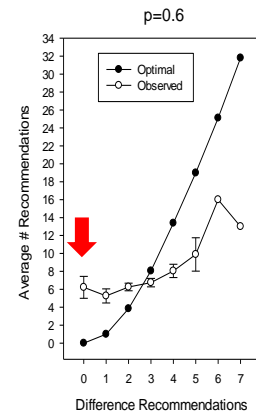


# Are people optimal decision makers?

(2) People terminated evidence collection when the critical difference was zero ( $d=0$ ; Pitz et al., 1969)

{Buy, Buy, Don't, Don't} → Buy

{1, 1, 0, 0} → 1



(3) People stopped on nondiagnostic patterns of recommendations

{Buy, Buy, Buy, Don't} → Buy

{1, 1, 1, 0} → 1

(4) The principle evidence against: decision making in small worlds

# Are people optimal decision makers?

People are not optimal decision makers.

Then people are suboptimal decision makers?

Heuristics

Bounded Rationality

Biased Decision Making

# A catalogue of boundedly rational stopping-rules

- Stopping rules:
  - (a) Fixed Number
  - (b) Critical Difference
  - (c) Runs
  - (d) Fixed forgetting model
  - (e) Horse Race or Accumulator Model
  - (f) Myopic Decision Rules.
- Although boundedly rational models have been able to explain some observed deviations from the optimal predictions no single such rule has been able to account for them all.
- The challenges:
  - No single rule can be used to explaining the total variance of observed behavior.
  - How to coordinate use of different stopping rules?
  - How to act adaptively in different environments?

# The Stopping Rule Selection (SRS) Theory

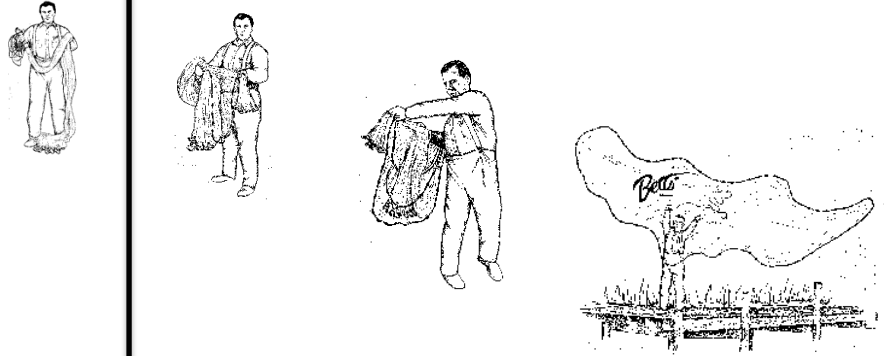
- The SRS theory provides the basis for a general approach to decision-making operations. This theory is consistent with the idea of a boundedly rational decision maker who utilizes simple decision rules in real time. In different environments, a decision maker acts adaptively, constantly looking for the best decision strategies, stopping rules, and critical values.

# A formal description of the SRS theory and proposed stopping rules.

- *Hypothesis 1: Multiple stopping rules.*
- *Hypothesis 2: Storage for stopping rules—the decision operative space (DOS).*
  - dimensionality reduction
- *Hypothesis 3: Retrieval of the stopping rules*

# Decision Operative Space (DOS)

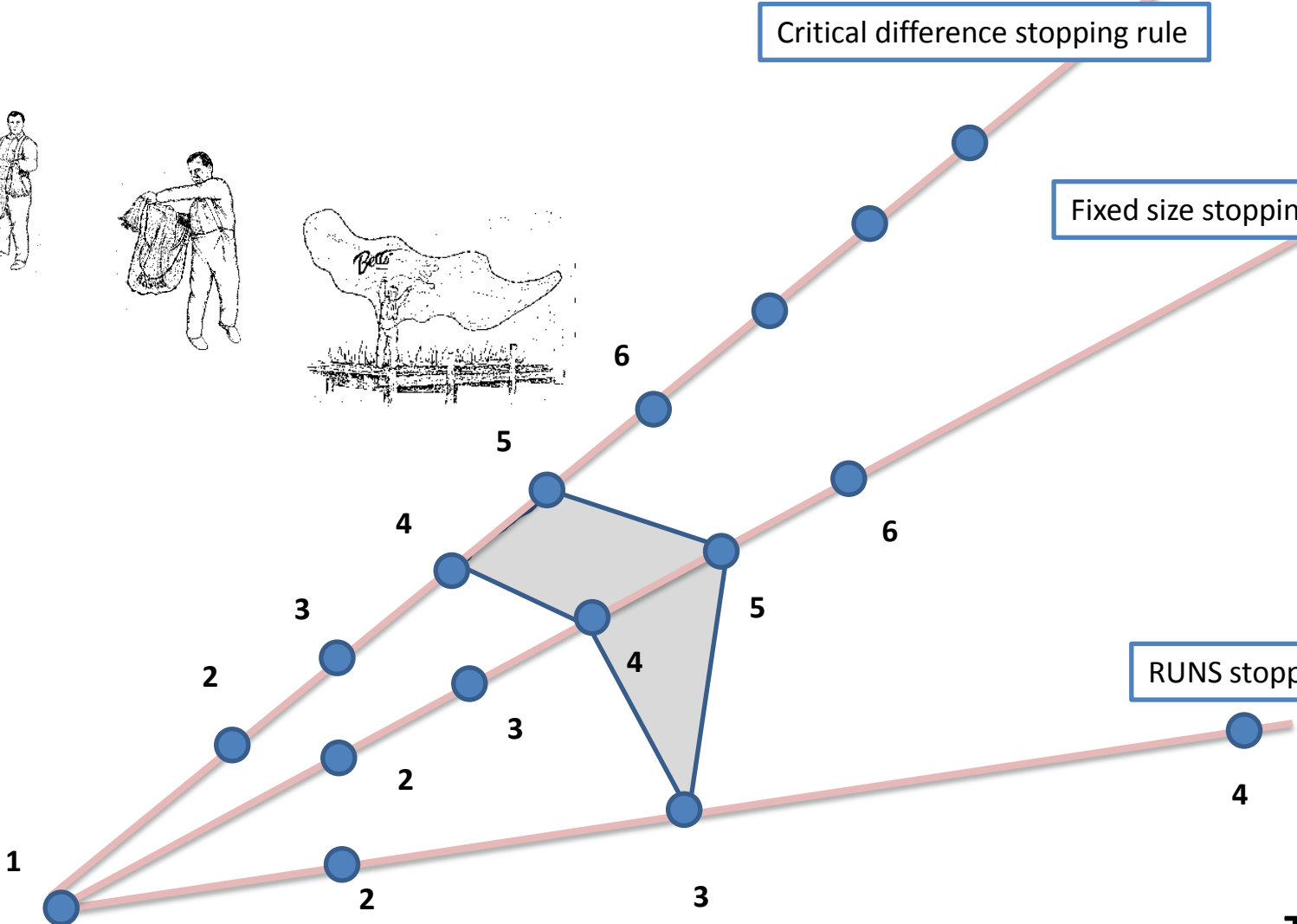
Cognitive Effort



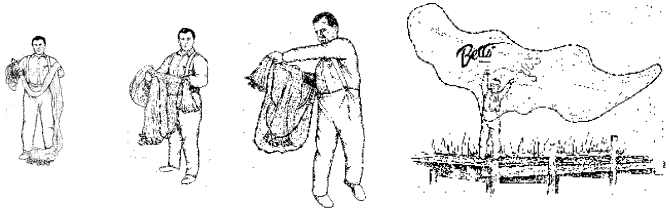
Critical difference stopping rule

Fixed size stopping rule

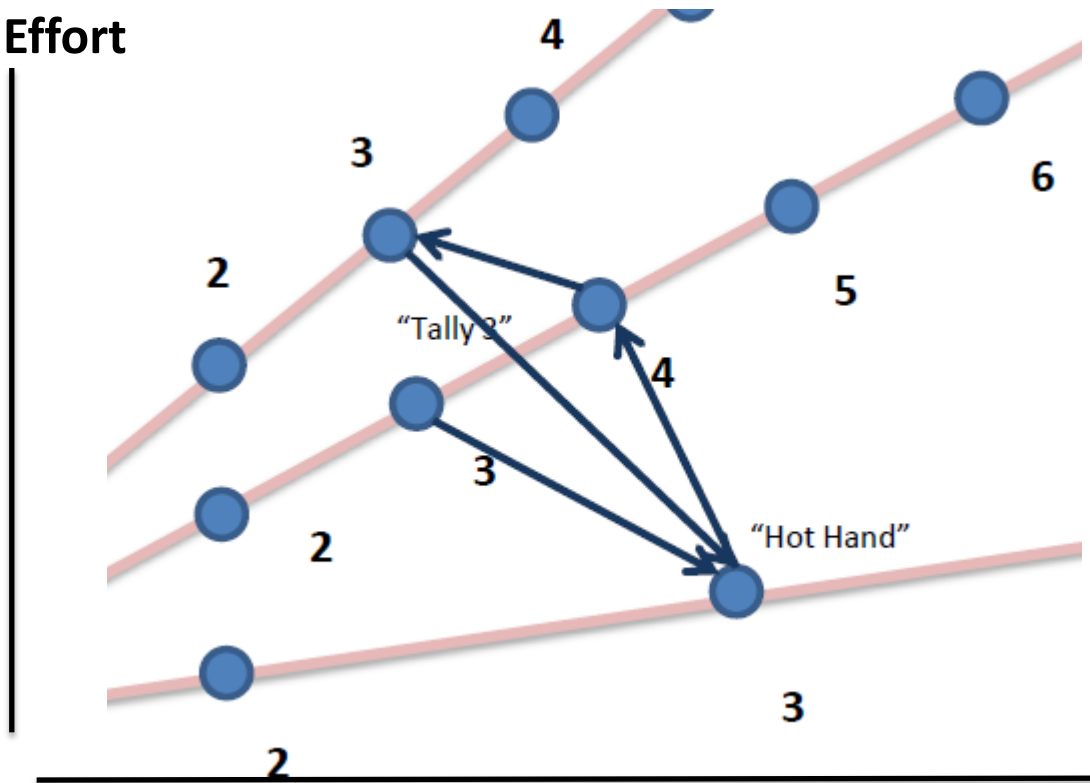
RUNS stopping rule



# Decision operative space

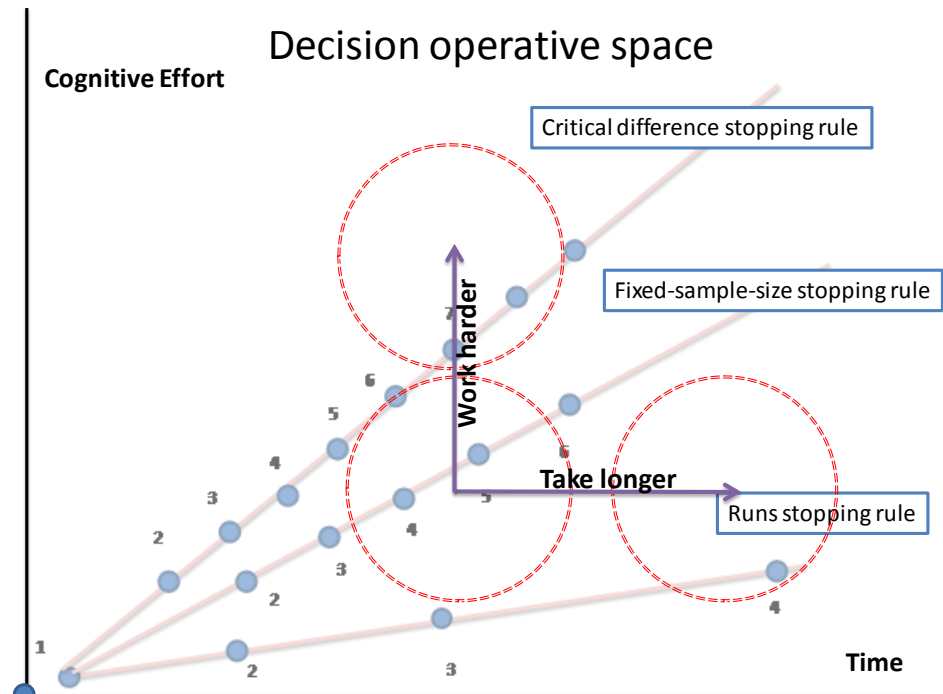
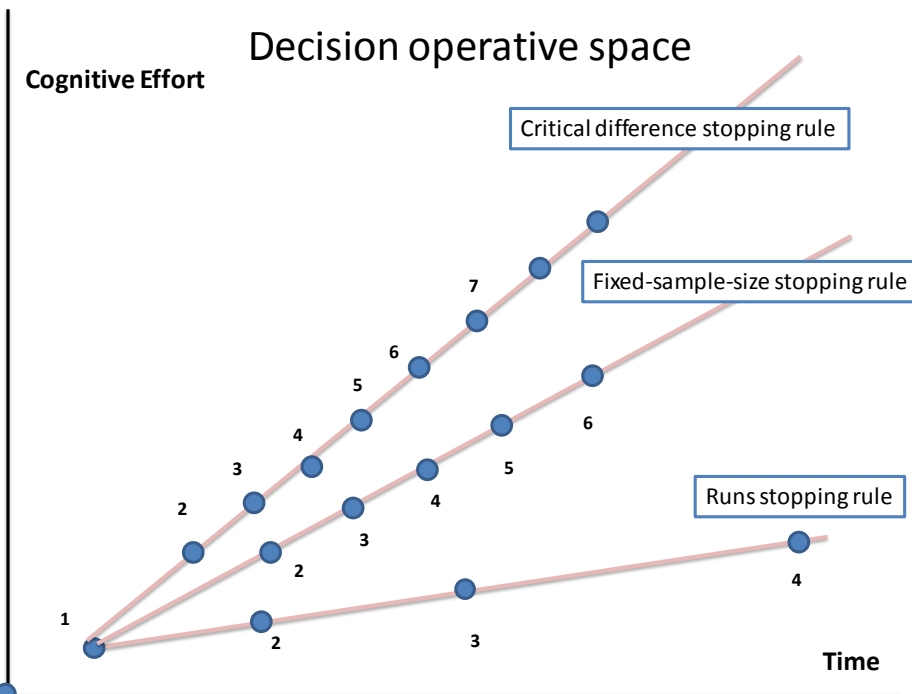


Cognitive Effort



# The *double* tradeoff

Depending on where stopping rules are retrieved from the DOS, a decision maker may choose to trade off **speed and accuracy** (cf. Diederich, 2003; Kocher & Sutter, 2006; Payne et al., 1993) or **cognitive effort and accuracy** (Payne et al., 1993).





# Validation: SRS model fit to Busemeyer and Rapoport (1988) data

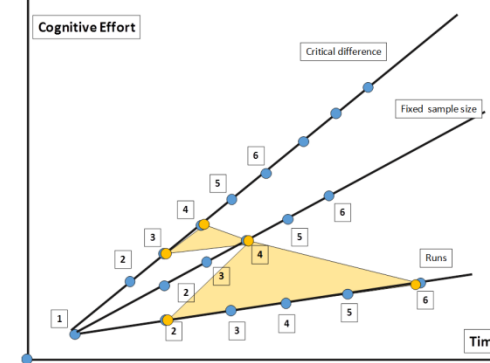
| Evidence                             | Response accuracy | Observed | SRS fit |
|--------------------------------------|-------------------|----------|---------|
| <b>Observed matched patterns</b>     |                   |          |         |
| {1, 1}                               | Correct           | 0.06     | 0.1     |
| {0, 0}                               | Correct           | 0.07     | 0.1     |
| {1, 1, 1}                            | Correct           | 0.19     | 0.17    |
| {0, 0, 0}                            | Correct           | 0.18     | 0.16    |
| {1, 0, 1, 1}                         | Correct           | 0.05     | 0.04    |
| {0, 1, 1, 1}                         | Correct           | 0.05     | 0.04    |
| {1, 1, 1, 1}                         | Correct           | 0.08     | 0.07    |
| {1, 1, 1, 0}*                        | Correct           | 0.001    | 0.01    |
| {1, 1, 0, 1}                         | Correct           | 0.05     | 0.03    |
| {1, 1, 0, 0}*                        | Incorrect         | 0.001    | 0.01    |
| {1, 0, 0, 0}                         | Correct           | 0.07     | 0.04    |
| {0, 0, 0, 0}                         | Correct           | 0.06     | 0.07    |
| {0, 1, 0, 0}                         | Correct           | 0.06     | 0.04    |
| {0, 0, 1, 0}                         | Correct           | 0.05     | 0.03    |
| {0, 0, 0, 1}                         | Correct           | 0.01     | 0.01    |
| <b>Observed non-matched patterns</b> |                   |          |         |
| {0, 0, 1}                            | Incorrect         | 0.002388 | 0       |
| {0, 1, 1}                            | Correct           | 0.009817 | 0       |
| {1, 0, 0}                            | Correct           | 0.002786 | 0       |



“1” = positive evidence opinion  
 “0” = negative evidence opinion

$R^2 = .86,$

A cast net spanned by 6 parameters



| Stopping rule       | Proportion recovered |
|---------------------|----------------------|
| Runs                | 0.25                 |
| Fixed sample size   | 0.46                 |
| Critical Difference | 0.29                 |
| Stop on one         | 0.00                 |

# Buying decision novel evidence

| Timing conditions | Recommendation |                | Observed matched patterns |          |          |           |
|-------------------|----------------|----------------|---------------------------|----------|----------|-----------|
|                   | Reliability    | R <sup>2</sup> | Evidence                  | Response | Observed | Predicted |
| Time pressure     | 0.6            | <b>0.73</b>    | {1.}                      | Correct  | 0.225    | 0.251     |
|                   | 0.75           | <b>0.79</b>    | {0.}                      | Inc      | 0.025    | 0.033     |
|                   | 0.9            | <b>0.91</b>    | {0.}                      | Correct  | 0.25     | 0.255     |
| Free-Evidence     | 0.6            | <b>0.46</b>    | {1.}                      | Inc      | 0.03125  | 0.0315    |
|                   | 0.75           | <b>0.57</b>    | {1.,1.}                   | Correct  | 0.1375   | 0.129     |
|                   | 0.9            | <b>0.81</b>    | {0.,0.}                   | Inc      | 0.00625  | 0.002     |
|                   |                |                | {0.,0.}                   | Correct  | 0.125    | 0.126     |
|                   |                |                | {1.,1.,1.}                | Correct  | 0.0375   | 0.0395    |
|                   |                |                | {0.,1.,1.}                | Correct  | 0.0125   | 0.005     |
|                   |                |                | {1.,0.,1.}                | Correct  | 0.00625  | 0.01      |
|                   |                |                | {0.,0.,0.}                | Correct  | 0.03125  | 0.0355    |
|                   |                |                | {0.,1.,0.}                | Correct  | 0.01875  | 0.015     |
|                   |                |                | {1.,0.,1.}                | Inc      | 0.00625  | 0.001     |
|                   |                |                | {1.,0.,1.,1.}             | Correct  | 0.01875  | 0.007     |
|                   |                |                | {1.,1.,0.,1.}             | Correct  | 0.00625  | 0.0015    |
|                   |                |                | {0.,1.,0.,0.}             | Correct  | 0.00625  | 0.0075    |
|                   |                |                | {1.,0.,0.,0.}             | Correct  | 0.00625  | 0.0065    |
|                   |                |                | {0.,0.,1.,0.}             | Correct  | 0.00625  | 0.003     |



# Conclusions

- The stopping rule selection theory (*SRST*) theory is consistent with the general idea of a **boundedly rational decision-making agent** utilizing simple decision rules, in real time.
- SRST does **not need a statistical description of the environment** for effective and adaptive decision making. In different environments, a decision maker acts adaptively, constantly looking for the best decision strategies, stopping rules, and critical values.
- The main innovation here is that for any given decision problem, a decision maker is not limited to using a single decision strategy or stopping rule but **can draw from a large set of different kinds of and stopping rules**.
- The SRST theory has shown a good fit of the challenging data. For this purpose we adapted ***pattern analysis***—an innovative method for fitting the relative frequency of observed data patterns to the predicted response patterns.
- The SRS model was able to account for the **three paradoxes** that falsified the optimal decision making approach.

# Benefits of SRST

- **Generality:** The SRS theory is more general than any single-stopping-rule evidence-accumulation model.
- **Parameter Space Reduction:** The cast-net approach can reduce the number of free parameters, despite the SRS theory being a general model.
- **Fast and Frugal:** The SRS model requires minimal computational effort compared to an optimal approach.
- **Corrective Feedback Is Not Needed:** The way in which the SRS theory chooses the stopping rules is determined solely by evaluation of the time and effort needed for the situation.
- **Stopping Is Guaranteed:** The mechanism of multiple stopping rules avoids the Buridan's donkey paradox. If the donkey had used a combination of the critical difference and the fixed-sample-size stopping rule, chances of stopping earlier would have increased, and the donkey would have been saved. Using multiple stopping rules is a very useful strategy in real life.
- **Saving Decision Time Through Statistical Facilitation:** If several stopping rules are used, all of which can result in the same decision accuracy, overall decision time will be faster than when only one stopping rule is used in isolation.