Teaching Supporting Material
for Tenure and Promotion to Associate Professor

Firas Hindeleh

MTH 201-07 **Calculus I** **Fall 2014**

Instructor: Dr. Firas Hindeleh
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Office Hours: TWF 10:00-11:00 (office), or by appointment.

Text: Active Calculus by Matt Boelkins. Textbook PDF-file is available on Blackboard.

Course Web Page: All announcements, assignments and other course details will be communicated through the course home page on Blackboard. Note that your userid and password are the same you use to access the campus network.

Prerequisites: MTH 122 and 123. You should be familiar with all the topics in Sections 1.1- 1.6 of our textbook. If you received less than a C+ in any of the math courses prior to Math 201, then you are at risk in this course and should schedule a meeting with me to discuss your plans for this class and your major.

General Education Category: MTH 201 fulfills Group IA (study of logical and Mathematical Quantitative Reasoning) in the university general education requirements.

Calculators: You are required to have a graphing utility for this class; carry it with you to class. Recommended is TI-83 or TI-84. You may use your favorite app on your smartphone with the understanding that I will not support you on how to use your app. When appropriate, I will use my TI-84 for demonstration in class. While I will explain *some* important commands, it is your responsibility to learn and interpret your calculator or mobile application.

Software: We will also use the mathematics software *Maple* as well as *Microsoft Excel* and some java applets to explore various aspects of Calculus. Introduction to important commands and features of these programs will be given in class.

Exams (30%): There will be three 50-minutes in class exams, tentatively scheduled
Exam 1: Tuesday, 9/23/14,
Exam 2: Tuesday, 10/28/2014,
Exam 3: Tuesday, 11/18/14.

Final Exam (15%): The final exam will be comprehensive and given on **Wednesday 12/10/2014, 12:00 - 1:50pm (MAK A-2-165).**

Labs, in class activities and Projects (25%): Throughout the course of the semester, there will be several in class activities to be done in groups, as well as lab activities. Some of those activities will be graded.

Preview Activities (10%): One of the unique features of the textbook is the set of Preview Activities at the beginning of each section. Students will complete these activities prior to the classroom discussion of each section. Each student can do them individually, but it is strongly recommended that students work in teams of two, three, or four students to complete these preview activities.

The purpose of the preview activities is to prepare the students for the classroom discussion of the section. It must be emphasized that it is permissible to make mistakes on the preview activities. In fact, the place to make mistakes (and then correct them) is on the preview activities.

Before the classroom discussion on a section, at least one of the preview activities for that section will be collected and graded. Grading will not be based on whether or not everything is correct, but rather on whether or not a serious and substantial effort was made to complete the Preview Activity. (Part of this effort is to write your answers in complete sentences using correct mathematical terminology and notation.) Each Preview Activity that is graded will be graded on a 4-point scale.

Attendance (2%): Attendance is critical to your success, and will be taken at the beginning of each class meeting. 50 points will be given for full attendance, 5 point off for every non excused absence (medical or athletic excuses must be given by a written document from the physician or coach). No other excuses will be accepted. You are responsible for the material covered, assignments, and changes in the syllabus.

Skills Test (3%): This is a 50-min in class exam that will be on 10/28/114. This test **must** be "passed" to earn credit for the course; the passing grade is "90%", there will be an opportunity to repeat this exam. Details will be provided later during the course.

Homework (15%): There will be weekly online homework using MAA's WebWork http://webwork-math.gvsu.edu/webwork2/MTH201-02 . You need to log in to webwork and complete the weekly homework. Your username is the prefix of your gvsu email (i.e for smithj@mail.gvsu.edu the username is smithj) and your password is the same as your GVSU network password.

Course Grade: Attaining the following numerical grades will ensure the associated letter grade: A 93%, A- 90%, B+ 87%, B 83%, B- 80%, C+ 77%, C 73%, C-70%, D+ 67%, D 63%, F <63%. Note that numerical grades **will not be rounded up**; for example, an 86.99% ensures a B.

Drop Date: The drop date for this course is Friday, October 24th, at 5:00pm.

Makeup exams: There will be no make up exams. If you miss an exam for a certifiable reason, other arrangements will be made. If you have to miss an exam, you MUST notify me beforehand; otherwise the grade is zero. The Final cannot be missed for any reason.

Math Center: The Mathematics Center is a place for students in many levels of math classes to get help. Check their address for the hours in the Spring at http://gvsu.edu/tutoring/math/

Student Concerns: If there is any student in this class who has special needs because of learning, physical, or other disability, please contact me or the Disability Support Recourses (DSR) at 331-2490.

Expectations for the Course

I will be using a flipped-classroom teaching style for this course. What this means is that you are required to read the assigned sections and solve the assigned preview activities prior to class. In class we will have an active discussion, and group activities. After class, you will need to check WebWork and attempt the assignment, and prepare for the next day. The most important rule for this course is that **you cannot get behind in your work!! Always stay up to date with the material in this course. Make succeeding in this course a priority** because this course is an important prerequisite for most upper level mathematics courses.

Attendance

Because this is a discussion-based course, attendance in class is critical to your success in this course. You are expected to be present and on time each day we meet. You are responsible for announcements made in class concerning material covered, assignments, and changes in the syllabus or due dates, or anything else pertinent to the course.

Preparation

It is imperative that you work on a consistent basis. This applies to both the day-to-day work to prepare for class as well as the more long-term work such as the research and writing assignment and the portfolio project. You should keep a well-organized record of your study notes, completed problems, and problems in progress for future reference. You must understand that a great deal of your learning in this course must occur on your own. It is your responsibility to read the text, do the problems, be prepared for class, and to seek help as needed.

Everyone must devote substantial time to carefully reading the textbook in order to come prepared to class to discuss the subject for the day. In every class meeting, every student will be expected to participate aloud at least once.

Participation

In every class meeting, there will be ample opportunity for you to actively participate. Through questions asked in lecture, brief exercises for small teams of students, and discussion of homework problems, you will be able to check and demonstrate your understanding of the material. You must be prepared and up to date to participate effectively. In addition, you will have the opportunity to ask questions in class. This is an important part of the learning process as I cannot respond to concepts that are causing you difficulty if you do not ask me about them. I expect that everyone will share in this important part of the learning process for this course.

Due Dates

All due dates for the course are strictly enforced. WebWork will automatically stop accepting attempts after the due date and time. Answers for the WebWork problems will be available shortly after the due date and time.

Honor System and Academic Honesty

It is expected that students will not have given nor received unauthorized aid in any work that is submitted for a grade in this course. Please refer to and carefully read the policy on academic honesty and plagiarism included at the end of this syllabus. Note well the penalties for such behavior in the course. On every assignment, I reserve the right to discuss the nature and origins of your work with you prior to awarding a grade on the work.

Graded Work and the Use of a Word Processor

I expect your very best work on all graded assignments. All course writing should adhere to the writing guidelines and principles that will be established in this course. For papers to be handed in, please use pen or pencil on loose-leaf paper, **stapled** if there is more than one page.

GENERAL EDUCATION
P R O G R A M

This course is part of GVSU's General Education Program.

The goal of the program is to prepare you for intelligent participation in public dialogues that consider the issues of humane living and responsible action in local, national, and global communities.

The program is designed to increase your knowledge and skills in the following areas:

Knowledge Goals

1. The major areas of human investigation and accomplishment - the arts, the humanities, the mathematical sciences, the natural sciences, and the social sciences.
2. An understanding of one's own culture and the cultures of others.
3. The tradition of humane inquiry that informs moral and ethical choices.

Skills goals

1. To engage in articulate expression through effective writing
2. To engage in articulate expression through effective speaking.
3. To think critically and creatively.
4. To locate, evaluate, and use information effectively.
5. To integrate different areas of knowledge and view ideas from multiple perspectives.

Ensuring that undergraduate students receive a broad general education has been a primary goal of colleges and universities since their inception. In this era of increasing specialization and growing demand for professional expertise, it is vital that we continue to emphasize the value of general learning.

GVSU maintains that a complete education involves more than preparation for a particular career. A career occurs in the context of a life, and a sound general education helps one "make a life" as well as "make a living." The university is committed to assuring that all undergraduate students, regardless of academic major, receive a broad education rooted in the arts and sciences.

Teaching in the liberal tradition is at the heart of Grand Valley's identity, and this focus is critical in our General Education Program. Liberal education transcends the acquisition of information; it goes beyond the factual to ask important evaluative and philosophical questions. Liberal learning holds the fundamental principles and suppositions of a body of knowledge up to inquiry, question, and discussion. It helps a person recognize the assumptions under which he or she operates and encourages the examination and questioning of those assumptions. Liberal learning begins in the General Education Program and continues through the more specialized studies comprising each student's major and minor areas of study.

Grand Valley State University educates students to shape their lives, their professions, and their societies.

MTH 201-07 (TR 11-1)
Fall 14
Tentative Schedule

Week 1		
Tuesday 8/26	Reading Prior Class:	Section 1.1
	Previews Due:	Prev. 1.1
	Screencasts:	http://bit.ly/1kzmXYv http://bit.ly/UNj99I http://bit.ly/1osDtc7
	In Class Activities:	Activities 1.1-3
Thursday 8/28	Reading Prior Class:	Section 1.2
	Previews Due:	Prev. 1.2
	Screencasts:	http://bit.ly/1pTIQhs http://bit.ly/1qMqeBJ http://bit.ly/1AOUfYd http://bit.ly/1xXLVBs
	In Class Activities:	Activities 1.4-6

Week 2		
Tuesday 9/2	Labor Day Recess	
Thursday 9/4	Reading Prior Class:	Section 1.3
	Previews Due:	Prev. 1.3
	Screencast:	http://bit.ly/UWH3iV http://bit.ly/1smX4Md http://bit.ly/1tO02LY
	In Class Activities:	Activities 1.7-9

Week 3		
Tuesday 9/9	Reading Prior Class:	Section 1.4
	Previews Due:	Prev. 1.4
	Screencasts:	http://bit.ly/UNkVaZ http://bit.ly/1zNKMPE http://bit.ly/1pzbC9M
	In Class Activities:	Activities 1.10-11
Thursday 9/11	Reading Prior Class:	Section 1.5
	Previews Due:	Prev. 1.5
	Screencasts:	http://bit.ly/1o6wpmI

		http://bit.ly/1nkQJed http://bit.ly/1smYd6E
	In Class Activities:	Activities 1.12-14

Week 4		
Tuesday 9/16	Reading Prior Class:	Section 1.6
	Previews Due:	Prev. 1.6
	Screencasts:	http://bit.ly/1zNLlsL http://bit.ly/1smYxSM http://bit.ly/WWARtk
	In Class Activities:	Activities 1.15-17
Thursday 9/18	Reading Prior Class:	Section 1.7
	Previews Due:	Prev. 1.7
	Screencasts:	http://bit.ly/1tO1Jcf http://bit.ly/UWIldV http://bit.ly/1AOVxTi http://bit.ly/1p4WT7h
	In Class Activities:	Activities 1.18-20

Week 5		
Tuesday 9/23	Reading Prior Class:	Section 1.8
	Previews Due:	Prev. 1.8
	Screencasts:	http://bit.ly/WWBoLI http://bit.ly/1tO2jGM http://bit.ly/1kzrFW5 http://bit.ly/1nkSnfK
	In Class Activities:	Activities 1.21
	Exam 1: Sections 1.1-1.7	
Thursday 9/25	Reading Prior Class:	Section 1.8
	Previews Due:	
	In Class Activities:	Activities 1.22

Week 6		
Tuesday 9/30	Reading Prior Class:	Section 2.1, 2.2
	Previews Due:	Prev. 2.1, 2.2
	Screencasts:	http://bit.ly/1qMv1CY http://bit.ly/1p4XplX http://bit.ly/1xXPwzp http://bit.ly/1kzsazs http://bit.ly/1xXPQhu
	In Class Activities:	Activities 2.1-6

Thursday 10/2	Reading Prior Class:	Section 2.3, 2.4
	Previews Due:	Prev. 2.3, 2.4
	Screencasts:	http://bit.ly/1smjbSg http://bit.ly/1osK5ak http://bit.ly/1lsNSA2 http://bit.ly/1qMw0TT http://bit.ly/1pTL28x http://bit.ly/1ufjoqz
	In Class Activities:	Activities 2.7-12

Week 7		
Tuesday 10/7	Reading Prior Class:	Section 2.5
	Previews Due:	Prev. 2.5
	Screencasts:	http://bit.ly/1mfAL5u http://bit.ly/1o6HfYQ http://bit.ly/1AOWFGj http://bit.ly/1lsOPbD http://bit.ly/1zNNV1P http://bit.ly/1xXRijZ http://bit.ly/1sn22Zz
	In Class Activities:	Activities 2.13-15
Thursday 10/9	Reading Prior Class:	Section 2.6
	Previews Due:	Prev. 2.6
	Screencasts:	http://bit.ly/UNpour http://bit.ly/1xXSJz1 http://bit.ly/1tO5Uoi
	In Class Activities:	Activities 2.16-18

Week 8		
Tuesday 10/14	Reading Prior Class:	Section 2.7
	Previews Due:	Prev. 2.7
	Screencasts:	http://bit.ly/1p4ZCxN http://bit.ly/1smmzwk http://bit.ly/1o6JBad
	In Class Activities:	Activities 2.19-20
	Skills Test	
Thursday 10/16	Reading Prior Class:	Section 2.7, 2.8
	Previews Due:	Prev. 2.8
	Screencasts:	http://bit.ly/1xXTfwS http://bit.ly/1zNPk8E http://bit.ly/1v27CE7

This exam has 4 questions, for a total of 100 points.
Answer the questions in the spaces provided on the question sheets showing all your work in detail.

Name : ______________________

1. For the function $f(x) = 3x^2 - 1$.

15 (a) Find $f'(0)$ using the limit definition. That is ...

$$f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$$

10 (b) Use your result in the previous part to find the equation of the tangent line at $x = 0$

2. A cup of coffee has its temperature F (in degrees Fahrenheit) at time t given by the function $F(t) = 75 + 110e^{-0.05t}$, where time is measured in minutes.

10 (a) Use a central difference with $h = 0 : 01$ to estimate the value of $F'(10)$. That is use the intervals $[9.99, 10]$ and $[10, 10.01]$.

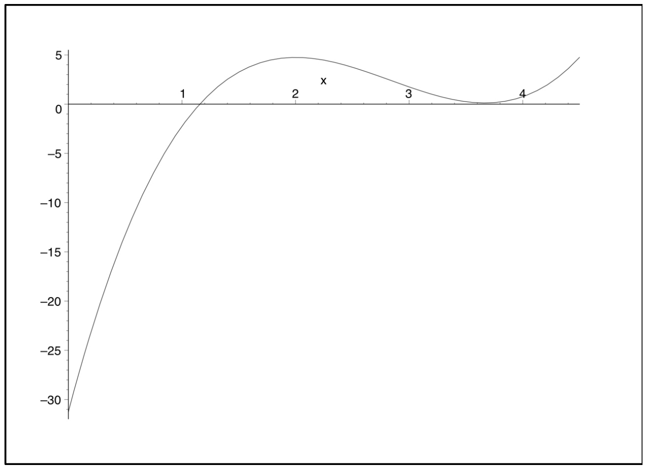
10 (b) What are the units on the value of $F'(10)$ that you computed in (a)? What is the practical meaning of the value of $F'(10)$?

10 (c) Which do you expect to be greater: $F'(10)$ or $F'(20)$? Why?

15 3. Draw the graph of a continuous function $y = f(x)$ that satisfy the following conditions:

- $f'(x) > 0$ for $-1 < x < 3$
- $f'(x) < 0$ for $x < -1$ and $x > 3$
- $f'(-1) = 0$ and $f'(3) = 0$

4. For an object moving along an axis with position $s(t)$ at time t, the object has the following **velocity** graph: Use this given graph of $v(t) = s'(t)$ to answer each question below.



10 (a) Give a point at which the particle's **position** decreasing? Justify your answer with a complete sentence.

10 (b) Give a point at which the particle's **velocity** increasing? Justify your answer with a complete sentence.

10 (c) Give a point at which the acceleration negative? Justify your answer with a complete sentence. [Hint: The acceleration is the derivative of the velocity, and the second derivative of the position.]

This exam has 7 questions, for a total of 150 points.
Answer the questions in the spaces provided on the question sheets showing all your work in detail.

Name : ______________________________

1. Given the following table, find the following showing all your work in detail..

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	2	3	1	-6
2	-2	4	5	7
3	4	6	-1	-3
4	-1	1	1	3

[5] (a) $\left(\frac{f}{g}\right)'(3)$

[5] (b) $(f \cdot g)'(2)$

[5] (c) $\left(f^{-1}\right)'(2)$

[3] (d) $\left(g^{-1}\right)'(1)$

[2] (e) $\left(g^{-1} \circ f^{-1}\right)'(2)$

2. Find the derivative of each of the following functions.

15 (a) $F(z) = e^{z^3+1} \cdot \arctan^4(z) + \ln(\sin(x))$

10 (b) $r(t) = \frac{t\ln(t)}{\cos(\sqrt{t})+t}$

3. Evaluate the following integrals. Do not use the calculator.

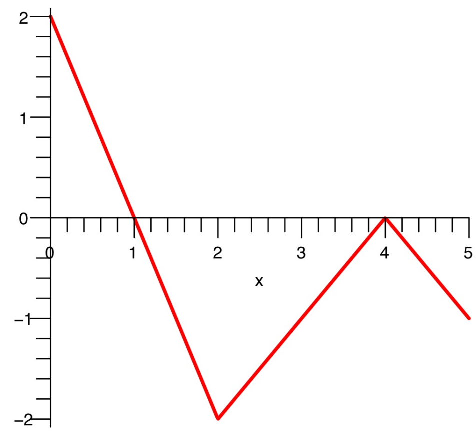
10 (a) $\displaystyle\int_1^2 \left(\frac{y^2-1}{\sqrt{y}}\right)^2 dy$

10 (b) $\displaystyle\int_0^{\pi} \left(\frac{x^3}{3} + 3x + 2e^x - \sin x + \cos x - \frac{3}{2}\right) dx$

4. Suppose that an object is moving along a straight line path has its velocity v (in meters per second) at time t (in seconds) given by the piecewise linear function whose graph is pictured below. Suppose further that the object's position at time $t = 1$ is $s(1) = 0$.

10 (a) Fill in the table showing your work. That is find the position of the object at time $t = 0, 2, 4$ and 5.

t	0	1	2	4	5
$s(t)$		0			



15 (b) At which intervals will $s'(t)$ and $s''(t)$ be positive or negative. Explain how did you figure them out.

5 (c) Graph s(t) based on the results obtained in the previous parts.

5. Find an estimate for area under the curve of $f(x) = 9 - x^2$ between $x = -1$ to $x = 2$ with $n = 3$ using

10 (a) Find Δx and fill the table below.

x	-1			2
$f(x)$				

10 (b) Left-hand sum.

5 (c) Find the exact area using your **calculator**.

15 6. A sailboat is sitting at rest near its dock. A rope attached to the bow of the boat is drawn in over a pulley that stands on a post on the end of the dock that is 5 feet higher than the bow. If the rope is being pulled in at a rate of 2 feet per second, how fast is the boat approaching the dock when the length of rope from bow to pulley is 13 feet?

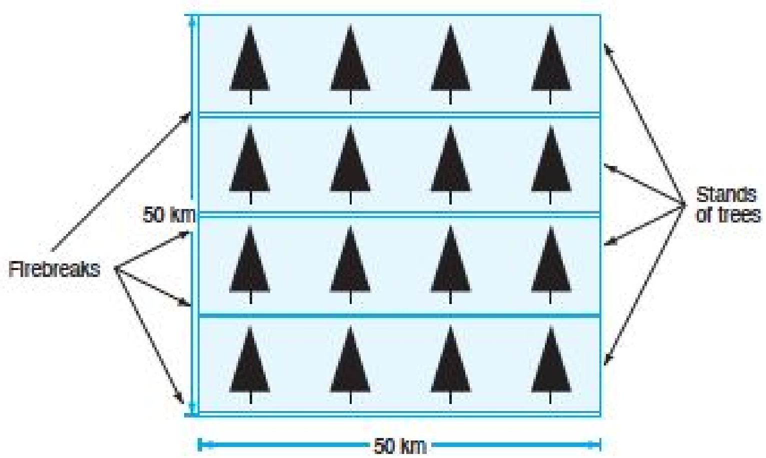
15 7. What are the dimensions of an aluminum can that holds 50in^3 of pop and that uses the least amount of material (i.e aluminum)? Assume that the can is cylindrical, and is capped on both ends. [A two-sided capped cylinder with radius r and hight h has a volume $V = \pi r^2 h$ and surface area $S = 2\pi r^2 + 2\pi r h$.]

This project has 1 questions, for a total of 50 points.
Write a paper that addresses the questions below. Incorporate the mathematics within your narration. You may work on this project in groups of up three. This is due Tuesday 11/25.

Name : ____________________

1. The summer of 2000 was devastating for forests in the western US: over 3.5 million acres of trees were lost to fires, making this the worst fire season in 30 years. This project studies a fire management technique called firebreaks, which reduce the damage done by forest fires. A firebreak is a strip where trees have been removed in a forest so that a fire started on one side of the strip will not spread to the other side. Having many firebreaks helps confine a fire to a small area. On the other hand, having too many firebreaks involves removing large swaths of trees.

30 (a) A forest in the shape of a 50 km by 50 km square has firebreaks in rectangular strips 50 km by 0.01 km. The trees between two firebreaks are called a stand of trees. All firebreaks in this forest are parallel to each other and to one edge of the forest, with the first firebreak at the edge of the forest. The firebreaks are evenly spaced throughout the forest. (For example, figure below shows four firebreaks.) The total area lost in the case of a fire is the area of the stand of trees in which the fire started plus the area of all the firebreaks.



(i). Find the number of firebreaks that minimizes the total area lost to the forest in the case of a fire.

(ii). If a firebreak is 50 km by b km, find the optimal number of firebreaks as a function of b. If the width, b, of a firebreak is quadrupled, how does the optimal number of firebreaks change?

20 (b) Now suppose firebreaks are arranged in two equally spaced sets of parallel lines, as shown in the figure below. The forest is a 50 km by 50 km square, and each firebreak is a rectangular strip 50 km by 0.01 km. Find the number of firebreaks in each direction that minimizes the total area lost to the forest in the case of a fire.

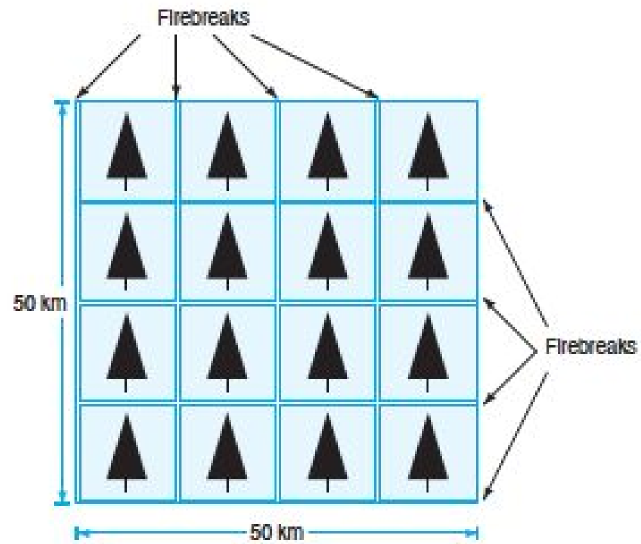


Figure 4.116

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Math 201
Lab 6: The Fundamental Theorem of Calculus

These exercises should be completed partly in groups of 2 to 3 students. More directions will follow from me.

Please explain all of your answers in detail and use complete sentences. You should write your solutions in this Maple document, using the Math mode to insert symbolic mathematical expressions appropriately.

This lab will help you to explore and investigate the meaning and significance of a result known as "the Second Fundamental Theorem of Calculus." The starting point for the exercises is a java applet that can be viewed using a web browser and going to

http://merganser.math.gvsu.edu/calculus/integration/ftc.html

(We thank Dr. David Austin of GVSU for the use of this applet.)

1. Explore the applet at the above URL by clicking, holding, and dragging the red dot on the left-hand curve. Carefully note what happens to the blue dot on the right-hand curve as you move the red one.

Write several sentences that carefully explain the relationship between the two curves, $f(x)$ and $G(x)$. There are many possible and important observations here; you should reply in as much depth and detail as possible. The focus of your response should involve the concept of "signed area"; at least one other sentence must use the word "derivative".

2. **Why** is the function $G(x)$ increasing on the (approximate) interval $0 < x < 2.1$? Explain your answer first using the idea of signed area; then do so using a property of the derivative of a function. Specific references must be made to the function $f(x)$. Conclude with a similar discussion regarding where $G(x)$ is decreasing.

3. **Where** does G have a relative maximum? A relative minimum? Why? How do these points correspond to important points on the graph of f?

4. What can be said about the behavior of the function $G(x)$ on intervals where the function $f(x)$ is decreasing? Why? Justify your answer with sentences that discuss both area and derivatives.

5. Note that at the top of the web page on which you are viewing the applet, you see that G is defined by the formula

$$G(x) = \int_0^x f(t)\, \mathrm{d}t$$

What does our work above tell us is true about $G(x)$? What implication does that have for us computing $\frac{d}{dx}\int_0^x f(t)\, \mathrm{d}t$?

6. Finally, based on the graphs, determine which familiar functions (i.e. ones with known, special names) are represented by $G(x)$ and $f(x)$. Write at least one sentence about the relationship between these two functions and why your choices make sense.

All 6 questions above are centered on the applet. The question below is a continuation of work begun in class with the given graph.

7. Below is given a graph of a function *f(t)*. Assume that for t values less than 0 and greater than 7, *f(t) = 0*. From the graph, construct an accurate, carefully labeled graph of an antiderivative F (of f) that satisfies $F(1) = 0$. Your graph should show F at least on the interval [-1,8]. In addition to your graph, include a formula for the antiderivative F.

Carefully justify your graph with appropriate calculations and writing to describe function values, increasing and decreasing behavior, and concavity. (This part should be done maually and not with Maple)

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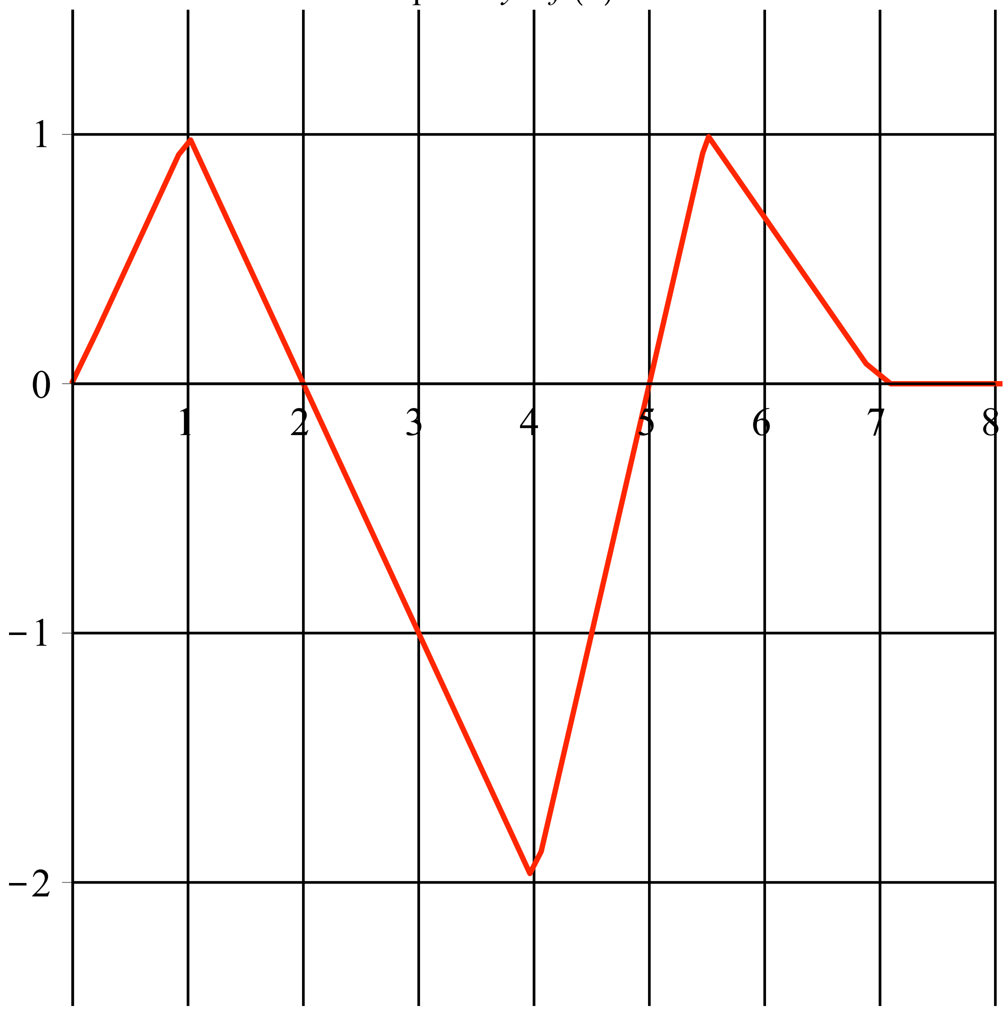
Graph of $y = f(t)$



Mathematics 227
Linear Algebra I

Section 01	T 4:00 - 5:15 P.M.	MAK A-2-151
Winter 2014	R 4:00 - 5:15 P.M.	MAK BLL 118

Instructor: Dr. Firas Hindeleh
MAK A-2-116
(616) 331-3739
hindelef@gvsu.edu
Office Hours: TWR 2:00 - 2:50 P.M.,
or by appointment

Prereqisites: Math 202

Text: *Linear Algebra and its Applications*, 3^{rd} edition, by David Lay

Course Content:

Systems of Linear Equations	Chapter 1
Matrices and their properties	Chapter 2
Determinants	Chapter 3
Vector Spaces	Chapter 4
Eigenvectors and Eigenvalues	Chapter 5

Grading: Your final grade will be determined in the following way:

Homework	25%
Quizzes	15%
Labs	15%
Exams	30%
Final Exam	15%

The final grade assigned to you will not be lower than that computed in this way. However, I reserve the right to raise your grade if you are an active, thoughtful participant in class and there is evidence you are working consistently and with care.

Homework: You will receive homework assignments every week. Homework is due at the beginning of Thursday's class. You are encouraged to work on your homework exercises in groups in which case one write up per group is sufficient. The importance of working through all the problems carefully cannot be overstated: working problems is the absolute best way to learn mathematics. For each homework assignment, you will be asked to write up solutions to a few problems carefully and submit them to me. These problems will be carefully marked and some feedback provided to you. You will be also be asked to hand in the rest of the assigned problems, which will be graded according to the effort that you have put into them.

Quizzes: There will a quiz at the beginning of Tuesday's class. Quizzes are based directly on the homework. In fact, if you work through the homework carefully, you will probably find the quizzes to be very straightforward. Quizzes will last approximately 10 minutes, so make sure not to be late to class.

Labs: We will meet once a week in the computer lab A-2-151 MAK. There we will have the opportunity to use technology to explore the course content in a different way. The work in the labs should be completed with a group of two or three students.

Exams: There will be three midterms scheduled for Feb 6, March 11, and April 3.
Note well: No makeup exams will be given. If you are unable to attend an exam, it is your responsibility to notify me *prior* to the exam so that suitable arrangements may be made.

Final Exam: The final exam will be Thursday, April 24 from 4:00 - 5:50 P.M in MAK BLL 118.

Final grade: Shown below is the percentage required to obtain a particular grade:

Grade	D	D-	C-	C	C+	B-	B	B+	A-	A
Percentage	60.0	67.0	70.0	73.0	77.0	80.0	83.0	87.0	90.0	93.0

Drop Date: The deadline for withdrawing from this course is Friday, March 7 at 5 P.M.

Expectations:

Attendance: Attendance is expected and critical to your success. Please be on time to class. You are responsible for all announcements made in class concerning material covered, assignments or anything else relevant to the course.

Academic Honesty: Some of the work in this course will be done collaboratively. It is also recommended that you form a study group to assist in learning the material and completing daily homework problems. However, it is important for you to understand that there is a difference between collaboration and plagiarism. Collaboration requires you to contribute to solutions and to think when you write. Representing someone else's work, no matter how small, as your own is plagiarism, a serious offense that will be met with a grade of zero and possible action under GVSU's guidelines. You are expected to show integrity in all your work and to encourage the same in your colleagues.

Mathematical Communication: One important part of mathematics is its emphasis on the clear and careful presentation of reasoning. This includes clearly stating the problem, making important observations in complete sentences, writing additional thoughts to clarify symbolic expressions, and showing a clear overall progression from problem to solution. The quality of your presentation and writing will count in all of your graded work.

Work load: To be successful in this course, you will need to work hard and consistently. A good rule of thumb is that you should spend at least two hours working outside of class for every hour of class time. Besides working on homework and other assignments, you should also keep a well-organized record of all your study notes and completed problems for future reference. In spite of the fact that we will discuss the most important concepts in detail during class, you will be expected to learn and assimilate many other ideas independently. Please take advantage of my office hours to discuss with me any problems you are having.

Participation: In every class meeting, you will be expected to participate actively and to share your understanding with the class. To do so effectively, you must come to class prepared. I expect that everyone will share in this important aspect of our learning process.

Late work: Late work will not be accepted. Any assignment is due on the stated date at the beginning of class.

Feedback: I am always happy to discuss with you any thoughts you have about the course including both your performance and mine. Please let me know if you think something could be better or if you like something that we are doing.

Student Concerns: Any student who requires accommodation because of a physical or learning disability must contact Disability Support Resources (`http://www.gvsu.edu/dsr`) at (616) 331-2490 as soon as possible. After you have documented your disability, please make an appointment or see me during office hours to discuss your specific needs.

Any student needing academic accommodations beyond those given to the entire class please be advised that the University's Office of Disability Support Services (DSR, ext. 12490) is available to all GVSU students. It is the student's responsibility to request assistance from DSR.

MTH 227-02 Test 3 April 8^{th}, 2010 **F. Hindeleh**

This test has 5 questions, for a total of 100 points.
Answer the questions in the spaces provided on the question sheets. **Show all your work in details**.

Name : ______________________________

1. Answer True or false. Justify your answer.

5 (a) If A and B are 2×2 matrices with columns $\mathbf{a_1}, \mathbf{a_2}$ and $\mathbf{b_1}, \mathbf{b_2}$, respectively, then $AB = [\ \mathbf{a_1}\mathbf{b_1} \quad \mathbf{a_2}\mathbf{b_2}\ ]$

5 (b) If A is an $n \times n$ matrix and the equation $A\mathbf{x} = \mathbf{0}$ has a non trivial solution, then A has fewer than n pivots.

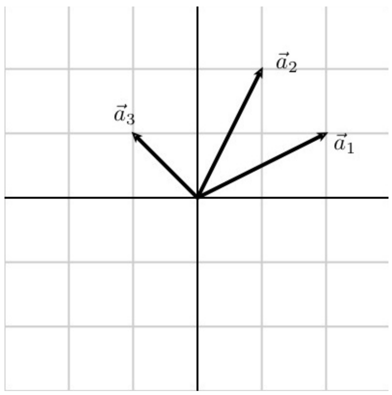
5 (c) If A is a 3×3 matrix with $|A| = 4$, then $|3A| = 12$.

5 (d) If A and B are $n \times n$ invertible matrices, then the inverse of AB is $A^{-1}B^{-1}$.

2. Suppose that $T : \mathbf{R}^3 \to \mathbf{R}^2$ is the linear transformation such that

$$T\left(\begin{bmatrix}1\\0\\0\end{bmatrix}\right) = \vec{a}_1, \qquad T\left(\begin{bmatrix}0\\1\\0\end{bmatrix}\right) = \vec{a}_2, \qquad T\left(\begin{bmatrix}0\\0\\1\end{bmatrix}\right) = \vec{a}_3$$

where $\vec{a}_1, \vec{a}_2$, and $\vec{a}_3$ are shown below (the grid is 1×1).



5 (a) Sketch the vector $T\left(\begin{bmatrix}-1\\0\\-2\end{bmatrix}\right)$ on the figure above.

5 (b) Find all vectors $\vec{x}$ whose image is the zero vector.

5 (c) Is T onto? Explain your thinking.

5 (d) Is T one-to-one? Explain your thinking.

3. Let A be

$$A = \begin{bmatrix} 2 & 0 & 0 & 2 \\ 0 & 0 & -2 & 2 \\ 1 & -2 & 2 & 1 \\ 7 & 0 & 0 & -5 \end{bmatrix}.$$

15 (a) Find $|A|$ **without** using the calculator.

5 (b) Find the solution set for the equation $A\vec{x} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$. Use your result from the previous part.

5 (c) Is A invertible? Explain your thinking.

5 (d) Find $|A^T \cdot A^{-1}|$

10 4. Use Cramer's rule to solve the following system $\begin{array}{rcrcr} 3x_1 & - & 2x_2 & = & 7 \\ -5x_1 & + & 6x_2 & = & -5 \end{array}$

20 5. Find the adjugate of the matrix $A = \begin{bmatrix} 0 & -2 & -1 \\ 3 & 0 & 0 \\ -1 & 1 & 1 \end{bmatrix}$, and use it to compute A^{-1}.

This test has 5 questions, for a total of 150 points.
Answer the questions in the spaces provided on the question sheets. **Show all your work in details**.

Name : ______________________________

1. Answer True or false. Justify your answer.

10 (a) If $\mathcal{B}$ and $\mathcal{C}$ are basis of a vector space V. Then the columns of the change of coordinate matrix $\underset{\mathcal{C} \leftarrow \mathcal{B}}{P}$ are the $\mathcal{B}-$coordinate vectors in the basis $\mathcal{C}$. That is

$$\underset{\mathcal{C} \leftarrow \mathcal{B}}{P} = \left[\begin{array}{cccc} [\mathbf{c}_1]_{\mathcal{B}} & [\mathbf{c}_2]_{\mathcal{B}} & \dots & [\mathbf{c}_n]_{\mathcal{B}} \end{array} \right]$$

10 (b) Every 2×2 diagonal matrix is invertible.

10 (c) If A is a $n \times n$ matrix then A is digonalizable if A has n eigenvalues counting multiplicity .

10 (d) The homogeneous system $A\mathbf{x} = \mathbf{0}$ has a non trivial solution if and only if $|A| = 0$.

15 2. Find the characteristic polynomial and the eigenvalues for $A = \begin{bmatrix} 5 & -3 \\ -4 & 3 \end{bmatrix}$

3. For $A = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 1 & 0 & 0 & 2 \end{bmatrix}$

10 (a) Find all eigenvalues and determine their multiplicity.

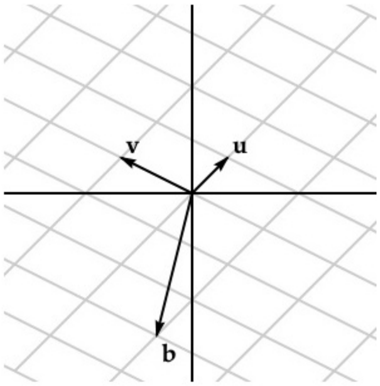
20 (b) For each eigenvalue in part (a), find the corresponding eigenspace.

10 (c) Is A dioganalizable? If so write A as the product of PDP^{-1}. i.e find P, D, P^{-1}

4. Suppose that a 2×2 matrix A defines a linear transformation T and that

$$T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \vec{u}, \qquad T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \vec{v}$$

where $\vec{u}$ and $\vec{v}$ are as shown below.



10 (a) Sketch on the diagram above the vector $T\left(\begin{bmatrix} 2 \\ 3 \end{bmatrix}\right)$. Explain below how you found it.

10 (b) Describe the solution set to the equation $A\vec{x} = \vec{b}$ where $\vec{b}$ is as shown above. Explain your response.

10 (c) Suppose that a linear transformation $S : \mathbf{R}^2 \to \mathbf{R}^2$ has

$$S\left(\begin{bmatrix} 1 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \text{and} \quad S\left(\begin{bmatrix} 2 \\ 7 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ 1 \end{bmatrix}. \quad \text{What is } S\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right)?$$

5. If $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & k \end{vmatrix} = 8$. Explaining your thinking, find

5 (a) $\begin{vmatrix} a & d & 5g \\ b & e & 5h \\ c & f & 5i \end{vmatrix}$

10 (b) $\begin{vmatrix} a & d & g+2a \\ b & e & h+2b \\ c & f & i+2c \end{vmatrix}$

10 (c) $\begin{vmatrix} e & b & h \\ d & a & g \\ f & c & k \end{vmatrix}$

Lab 7

Hindeleh

Matrix Lie algebra Change of basis

Refere to the presentation handouts for Lie algebra defintions.

Define the commutator $[A, B] = AB - BA$ by the function
$Com := (A, B) \rightarrow A.B - B.A$

$$(A, B) \rightarrow Typesetting{:}\text{-}delayDotProduct(A, B) - Typesetting{:}\text{-}delayDotProduct(B, A) \qquad \textbf{(1)}$$

The first Lie algebra of dimension 4 (called A4.1) is spanned my the basis $\{M1, M2, M3, M4\}$ where the M's are the following 4x4 matrices

$M1 := [[0, 0, 0, 1], [0, 0, 0, 0], [0, 0, 0, 0], [0, 0, 0, 0]]$	$M2 := [[0, 0, 0, 0], [0, 0, 0, 1], [0, 0, 0, 0], [0, 0, 0, 0]]$
$M3 := [[0, 0, 0, 0], [0, 0, 0, 0], [0, 0, 0, 1], [0, 0, 0, 0]]$	$M4 := [[0, 1, 0, 0], [0, 0, 1, 0], [0, 0, 0, 0], [0, 0, 0, 0]]$

- Find ALL possible bracket relations using the function above.

- Let $\{S1, S2, S3, S4\}$ be another basis for the same Lie algebra

A4.1 where you know the following:

$$M1 = \frac{1}{10}S1 - \frac{1}{10}S3 - \frac{1}{10}S4$$

$$M2 = \frac{1}{10}S1 + \frac{1}{5}S2 + \frac{1}{10}S4$$

$$M13 = \frac{1}{5}S1 + \frac{1}{10}S2 + \frac{1}{10}S3 + \frac{1}{10}S4$$

$$M4 = \frac{1}{10}S1 + \frac{1}{10}S4$$

Which change of basis matrix does this information give you, $\underset{M \leftarrow S}{P}$ or $\underset{S \leftarrow M}{P}$?

Define the appropriate ($\underset{M \leftarrow S}{P}$ or $\underset{S \leftarrow M}{P}$) matrix and use it to find the other one.

- Find the 4 matrices $\{S1, S2, S3, S4\}$.

- Assuming that you are given the values of the four matrices $\{S1, S2, S3, S4\}$ in the previous part and the value of the four matrices $\{M1, M2, M3, M4\}$given at the very beginning of this activity. Find $\underset{S \leftarrow M}{P}$by solving for the columns of this matrix. Does

GRAND VALLEY STATE UNIVERSITY
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MTH 210-01 SWS Communicating in Math Winter 13

Instructor: Dr. Firas Hindeleh
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Email: hindelef@gvsu.edu, MTH210FH@gmail.com
Twitter: @firashindeleh

Office Hours: MF 11-11:50, R 10-10:50 (office), or by appointment.

Text: *Mathematical Reasoning: Writing and Proof, 3rd Edition, by Ted Sundstrom.* Course pack can be purchased from the bookstore.

Course Web Page: All announcements, assignments and other course details will be communicated through the course home page on Blackboard. Note that your userid and password are the same you use to access the campus network.

Prerequisites: MTH 201 and WRT 150.

Supplemental Writing Skills: MTH 210 is a designated SWS (Supplemental Writing Skills) as described in the GVSU catalog. Completion of Writing 150 with a grade of C or better (not C-) is a prerequisite. SWS credit will not be given to a student who completes this course before completing Writing 150. SWS courses adhere to certain guidelines. Students turn it a total of at least 3000 words of writing during the semester. Part of that total may be essay exams, but a substantial amount of it is made up of finished essays or reports or research papers. The instructor works with the students on revising drafts of their papers, rather than simply grading the finished pieces of writing. At least four hours of class time are devoted to writing instruction. At least one-third of the final grade is based on the writing assignments.

Internet Access and Student email: Most of the materials and information for this course will be posted to Blackboard's course page. The Internet address for the GVSU Blackboard System is http://mybb.gvsu.edu .
Students are expected to check the course home page daily since the course schedule and assignments will be posted on this home page. Students are also expected to use the e-mail provided by GVSU, as the instructor will frequently send e-mail messages to the entire class.

Course Content: Elementary logic, sets, axiomatic systems, elementary number theory, relations, functions, and methods of mathematical proof including direct proofs, indirect proofs, mathematical induction, case analysis, and counterexamples.

Course Objectives:

- To develop logical thinking skills and to develop the ability to think more abstractly in a proof oriented setting.

- To develop the ability to construct and write mathematical proofs using standard methods of mathematical proof including direct proofs, mathematical induction, case analysis, and counterexamples.
- To develop the ability to read and understand written mathematical proofs.
- To develop talents for creative thinking and problem solving.
- To improve the quality of communication in mathematics. This includes improving writing techniques, reading comprehension, and oral communication in mathematics.
- To explore and understand the concepts described in the course content above.

Drop Date: The drop date for this course is Friday, March 8th, at 5:00pm.

Student Concerns: GVSU seeks to ensure that its programs are accessible to all persons. Students in need of special assistance or an accommodation regarding any of the course requirements as outlined in this syllabus, the course objectives, and/or course evaluation and assessment criteria, are advised to notify the Disability Support Resources (200 STU, 616-331-2490) within the first two weeks of class.

Any student who requires accommodation because of a physical or learning disability must contact Disability Support Recourses (http://www.gvsu.edu/dsr) at 616-331-2490 as soon as possible. After you have documented your disability, please make an appointment or see me during office hours to discuss your specific needs. It is the student's responsibility to request assistance from DSS.

Grading for the Course

Homework

Daily reading and exercises will be made and homework will be reviewed in class. These homework assignments will not be collected.

Preview Activities (10% of the course grade)

One of the unique features of the textbook is the set of Preview Activities at the beginning of each section. Students will complete these activities prior to the classroom discussion of each section. Each student can do them individually, but it is strongly recommended that students work in teams of two, three, or four students to complete these preview activities.

The purpose of the preview activities is to prepare the students for the classroom discussion of the section. It must be emphasized that it is permissible to make mistakes on the preview activities. In fact, the place to make mistakes (and then correct them) is on the preview activities.

Before the classroom discussion on a section, at least one of the preview activities for that section will be collected and graded. Grading will not be based on whether or not everything is correct, but rather on whether or not a **serious and substantial effort** was made to complete the Preview Activity. (Part of this effort is to write your answers in complete sentences using correct mathematical terminology and notation.) Each Preview Activity that is graded will be graded on a 4-point scale.

Assignments (10% of the course grade)
During the second half of the semester, there will be 2 team assignments. These assignments will be take-home assignments that will be distributed in class on a Monday and will be due on the following Monday. For each assignment, each team must turn in one completed assignment. By including the names of all team members, the team is stating that each member participated significantly in the development of the conjectures and proofs in the assignment and everyone on the team understands the proofs. The write-ups will be distributed randomly to other teams for feedback.

Mid-Term Examinations (30% of the course grade)
A mid-term examination will be given in two parts. Each part will be graded on a 100-point scale. Part I is tentatively scheduled for **Monday 2/11/2013** and Part II is tentatively scheduled for **Wednesday 3/27/2013**. There may be a take-home portion for this exam. If so, the take-home portion will be due right before the exam starts. No make-up exams will be given without permission from the instructor prior to the date of the exam.

Portfolio Project (35% of the course grade)
You will turn in ten mathematical problems from 3 groups of problems that will be included in a "MTH 210 Portfolio." You may turn in each of these problems and essay two times to be critiqued. After each submission, you have the opportunity to rewrite your portfolio proof or essay prior to submission for a final grade. However, *no more than two problems may be submitted for review on a given day, and no more than four problems (or essay) may be submitted for review during any week*. Do not delay working on those problem till before the due date.

The portfolio will be graded on a 110 point scale. Each problem will be worth 10 points (for a total of 100 points), and in addition, there will be 10 points possible for submission of drafts for review by the professor. Detailed information and due dates are included in the "*Guidelines for the Portfolio Project*" document.

Final Examination (15% of the course grade)
The final examination will be a comprehensive test that will be graded on a 200 points scale. The final exam is scheduled for Tuesday April 23, 2013 from 12:00 -1:50 pm in (B-1-122 MAK). There will be a take home portion that will be due on 4/23/2013 at 12:00pm.

Course Grade:
Attaining the following numerical grades will ensure the associated letter grade: A 93%, A- 90%, B+ 87%, B 83%, B- 80%, C+ 77%, C 73%, C-70%, D+ 67%, D 63%, F <63%. Note that numerical grades **will not** be rounded up; for example, an 86.9% ensures a B.

Expectations for the Course

This is the Supplemental Writing Skills course for the mathematics major. You will be expected to spend a great deal of time writing and rewriting your essays, assignments, and problems in your portfolio project, and you will be responsible for the mathematical content in this course. The most important rule for this course is that **you cannot get behind in your work!! Always stay up to date with the material in this course. Make succeeding in this course a priority** because this course is an important prerequisite for most upper level mathematics courses.

Attendance

Because this is a discussion-based course, attendance in class is critical to your success in this course. You are expected to be present and on time each day we meet. You are responsible for announcements made in class concerning material covered, assignments, and changes in the syllabus or due dates, or anything else pertinent to the course.

Preparation

It is imperative that you work on a consistent basis. This applies to both the day-to-day work to prepare for class as well as the more long-term work such as the research and writing assignment and the portfolio project. You should keep a well-organized record of your study notes, completed problems, and problems in progress for future reference. You must understand that a great deal of your learning in this course must occur on your own. It is your responsibility to read the text, do the problems, be prepared for class, and to seek help as needed.

Everyone must devote substantial time to carefully reading the textbook in order to come prepared to class to discuss the subject for the day. In every class meeting, every student will be expected to participate aloud at least once.

Participation

In every class meeting, there will be ample opportunity for you to actively participate. Through questions asked in lecture, brief exercises for small teams of students, and discussion of homework problems, you will be able to check and demonstrate your understanding of the material. You must be prepared and up to date to participate effectively. In addition, you will have the opportunity to ask questions in class. This is an important part of the learning process as I cannot respond to concepts that are causing you difficulty if you do not ask me about them. I expect that everyone will share in this important part of the learning process for this course.

Due Dates

All due dates for the course are strictly enforced. It is expected that all assignments will be turned in before midnight of the due date. No late work will be accepted without prior approval from the instructor.

Honor System and Academic Honesty

It is expected that students will not have given nor received unauthorized aid in any work that is submitted for a grade in this course. Please refer to and carefully read the policy on academic honesty and plagiarism included at the end of this syllabus. Note well the penalties for such behavior in the course. On every assignment, I reserve the right to discuss the nature and origins of your work with you prior to awarding a grade on the work.

Graded Work and the Use of a Word Processor

I expect your very best work on all graded assignments. All course writing should adhere to the writing guidelines and principles that will be established in this course. For papers to be handed in, please use pen or pencil on loose-leaf paper, stapled if there is more than one page.

All solutions for the problems on the Portfolio Project and all drafts of the research assignment must be written on a word processor capable of producing the appropriate mathematical symbols and equations. Microsoft Word and its Equation Editor, which is available on the student network, is one such word processor. Use 12 point font and standard margins. All such papers must be spell-checked and proofread for grammatical correctness.

Mathematical Writing

One of the primary objectives of this course is to learn to write mathematics well and in particular, learn to write mathematical proofs. Writing is an important part of communicating mathematical results. You will be required to write mathematical proofs and solutions to mathematical problems on the tests and assignments for this course. Writing proofs and solutions means more than writing formulas and circling an answer. It requires explanations of all significant steps taken in the solution of a problem. These explanations must be written in complete sentences and paragraphs with appropriate formulas and graphs included. The grading of the writing assignments will be based on the quality of the writing, the quality of the mathematical content, and the logical organization of the writing.

Academic Honesty and Plagiarism Policy

One of the primary goals in this capstone course is to get you to ``think like a mathematician." Explicitly, the point is for you to deepen your own understanding of mathematics and to gain individual appreciation and insight regarding the nature of mathematics.

As such, all of the work that you complete as part of the graded requirements for this course must be your own.   In everything that you write, I will be looking to find your personal understanding and development in the course of studying the material. To be clear, I have no interest in you emulating the work of one of your classmates, replicating the efforts of a student from a prior semester in MTH 210, nor in work taken from an external resource such as a textbook or Internet site. All of this is to say that your work must be completed with the highest level of academic honesty and integrity, and that plagiarism will not be tolerated.

This document establishes our guidelines for the semester regarding academic honesty and plagiarism, hopefully setting appropriate boundaries for each student so that we can achieve the goals stated above for personal learning and understanding. This policy is in effect for all students in MTH 210 for the duration of the term. Please be sure to read it carefully and to honor it accordingly.

> **Plagiarism** is the act of submitting the work of someone else as if it were your own. Specifically, this action intends to mislead the instructor to think that the work is the result of learning accomplished by the student named on the paper.

While there are many terrible things about plagiarism, the worst may be that committing the act once will call into question all of your work in a course. In addition, in an environment where students engage in academic dishonesty, the instructor is forced to look at everyone's work and question it. This is particularly unfair to the students who are doing honest work.

The following are guidelines for avoiding plagiarism in course assignments. The list is representative, but not exhaustive. Evidence of such behavior on any assignment will be grounds for a minimum penalty of zero on the entire assignment. In severe cases, the penalty will be failure of the course. In all cases, the guidelines established in the GVSU catalog and GVSU student code will be followed. I reserve the right to discuss the nature and origins of any assignment with you prior to awarding a grade on the paper.

1. On the assignments, every sentence that you write should be one that you have generated yourself and that you understand. While you are permitted to collaborate on big ideas and hints on problems with classmates, you must be working alone when you write your solutions. All collaboration with homework problems must occur with students in this class who are currently at the same stage of problem solution as you.

To be clear, suppose that you asked 4 different students in the class "How did you do problem 4?'" You did this at a time when you had made no personal progress on the problem, and you asked until you found someone who both had the problem completed and was willing to give you a route to the solution. Such an act constitutes plagiarism, for the work is simply not your own. On the other hand, it is entirely fine to work with one or two peers who are similarly stuck and to "put your heads together."

Do note that your instructor is generous with hints and is always willing to discuss problems with you. While I will never simply give you the answer, I will offer direction and guidance that will assist you in coming to a solution on your own.

2. On the assignments, the primary resource you should use is the course textbook. If you look up relevant review material from a past mathematics course (e.g. calculus), you should cite the book you used and the specific pages you considered. You are not, however, permitted to go looking for completed solutions to current homework problems in any other texts or resources.

   In particular, using the Internet is completely off limits unless specifically allowed on a particular assignment. Evidence of using Internet sources in your work will result in a minimum penalty of failure of the assignment.

3. On any assignment, it is an act of plagiarism to base your work on the efforts of a friend or acquaintance that has completed the course in a prior term. Be advised that in many instances, other instructors have kept copies of essays, homework assignments, and semester projects and I will be able to access them. I am well aware that students often share past exams, homework assignments, and more with one another. But, such sharing strives to defeat the point of the course for those of you registered this term, and therefore is not permitted. If you have any such materials in your possession, please return them immediately to their rightful owner. Use of such materials in your work this semester is grounds for failure of the course.

Again, the entire point of any course is for you to learn, grow, and mature as a student. Such development can only happen in an environment of academic honesty. I expect that each of you will adhere to these guidelines and that you will play active roles in encouraging your peers to do likewise. At any time, you are welcome to discuss with me questions or concerns that you have regarding this policy, your own work, or the work of your peers.

1. Mon 1/7	Previews Due:	None
	In Class:	Introduction to the Course, Discuss Sec 1.1
	Screencasts:	Statements and Non-Statements (Screencast 1.1.1) http://youtu.be/UuETUEJo0rk How do we know if a statement is true? (Screencast 1.1.2) http://youtu.be/z-TPb8hI58k Conditional statements (Screencast 1.1.3) http://youtu.be/1f2l2t4MAwk When are conditional statements true? (Screencast 1.1.4) http://youtu.be/9rz_iC2t0iE Truth tables for conditional statements (Screencast 1.1.5) http://youtu.be/iT1FgtoeFx4
	Homework:	Section 1.1: 1, 2
Wed 1/9	Previews Due:	Section 1.2
	In Class:	Complete Sec 1.1, Start 1.2
	Screencasts:	Working with definitions (Screencast 1.2.1) http://youtu.be/RoqoSxUoK-A Working with definitions, part 2 (Screencast 1.2.1b) http://youtu.be/u-itAHieTfI
	Homework:	Section 1.1: 3, 4, 5, 6
Fri 1/11	Previews Due:	None
	In Class:	Complte Sec 1.2
	Screencasts:	Direct proofs of conditional statements using know-show tables (part 1) (Screencast 1.2.2) http://youtu.be/H8LLINU6ebY Direct proofs of conditional statements using know-show tables, Part 2 (Screencast 1.2.3) http://youtu.be/1tCOucLfdh0 Writing up a proof outline into a paragraph (Screencast 1.2.4) http://youtu.be/55p4Ehdl7AY
	Homework:	Section 1.2: 1(a), 2, 3(c), 4, 5(a), 6, 9, 10(a), 11

2. Mon 1/14	Previews Due:	Section 2.1
	In Class:	Discuss Section 2.1
	Screencasts:	Negations of simple statements (Screencast 2.1.1) http://youtu.be/PG6zHqt9PsI Truth tables, part 1 (Screencast 2.1.2) http://youtu.be/d2yksDk4h6s Truth tables, part 2 (Screencast 2.1.3) http://youtu.be/GKRryCRG4Tk Truth tables, part 3 (Screencast 2.1.4) http://youtu.be/0j0DsAWbP7E Truth tables, part 4 (Screencast 2.1.5) http://youtu.be/_GD25pbPTFs Truth tables, part 5 (Screencast 2.1.6) http://youtu.be/bJzDrQGQEpM Tautologies and contraditions, part 1 (Screencast 2.1.7) http://youtu.be/w6_Yldevy5s Tautologies and contradictions, part 2 (Screencast 2.1.8) http://youtu.be/8nB1e64CzJM
	Homework:	Section 2.1 – 1, 3, 4, 5
Wed 1/16	Previews Due:	Section 2.2
	In Class:	Discuss Section 2.2
	Screencasts:	Logical equivalence (Screencast 2.2.1) http://bit.ly/NH6WcA Converse and contrapositive http://bit.ly/QK9AV5 Negation of Conditional Statements http://bit.ly/TFaXRt Logical equivalencies without truth tables http://bit.ly/PVeo5W
	Homework:	Section 2.2 - 1, 2, 3, 5, 6, 7, 8

Fri 1/18	Previews Due:	Section 2.3
	In Class:	Discuss Section 2.3
	Screencasts:	Sets and set notation http://bit.ly/Q9KpHw Open sentences and truth sets http://bit.ly/RU7jBM Elements, subsets, and set equality http://bit.ly/RU7RYu Set-builder notation http://bit.ly/PPnBwp
	Homework:	Section 2.3 – 1, 2, 3, 5, 6
3. Mon 1/21	MLK day: No classes	
Wed 1/23	Previews Due:	Section 2.4
	In Class:	Discuss Section 2.4
	Screencasts:	Quantified statements **http:**//bit.ly/NZCvP4 Negating quantified statements http://bit.ly/SqRcSH
	Homework:	Section 2.4 – 1, 2(a, c, d, f, g), 3(b, c, f), 4(b, c, f)
4. Mon 1/28	Previews Due:	Section 3.1
	In Class:	Discuss Section 3.1
	Screencasts:	Integer divisibility http://**bit.ly/P8VJpU** Direct proof involving divisibility **http://bit.ly/PPqbSZ** Integer congruence http://bit.ly/RUb8XK
	Homework:	Section 3.1 – 1, 2, 4, 7

4. Wed 1/30	Previews Due:	None
	In Class:	Continue with 3.1
	Screencasts:	Reducing an integer modulo n http://bit.ly/QSxjNC Proofs involving integer congruence. http://bit.ly/PjptwE
	Homework:	Section 3.1 –8, 11, 15
Friday 2/1	Previews Due:	Section 3.2
	In Class:	Discuss Section 3.2
	Screencasts:	http://bit.ly/QqpnqM http://bit.ly/QIApFz http://bit.ly/PQMkzW
	Homework:	Section 3.2 – 2, 3, 5
Fri 9/21	Previews Due:	None
	In Class:	Continue 3.2
	Screencasts:	http://bit.ly/SclWVt http://bit.ly/NHjqpj
	Homework:	Section 3.2 – 8, 9, 11,14, 18

5. Mon 2/4	Previews Due:	Section 3.3
	In Class:	Discuss Section 3.3
	Screencasts:	http://bit.ly/Pvn5F6 http://bit.ly/NHiGRc
	Homework:	Section 3.3: 1, 2, 3
Wed 2/6	Previews Due:	None
	In Class:	Continue Section 3.3
	Screencasts:	http://bit.ly/Sclruy http://bit.ly/OJNWQN
	Homework:	Section 3.3: 9, 12, 15, 16
Fri 2/8	Previews Due:	Section 3.4
	In Class:	Discuss Section 3.4
	Screencasts:	Proof by cases Part 1 http://bit.ly/TCdceb Proof by cases Part 2 http://bit.ly/Oqe17O Proof by cases Part 3 http://bit.ly/S7XuAx Proof by cases Part 4

Directions

1. **For each conjecture**, determine whether the conjecture is true or false. If the conjecture is true, state an appropriate theorem or proposition and prove it. If a conjecture is not true, provide a counterexample to show that it is false. In addition:
   - If a **biconditional statement** is found to be false, you should clearly determine if one of the conditional statements within it is true. In that case, you should state an appropriate theorem for this conditional statement and prove it.
   - If you determine that a conjecture to prove that that **two sets are equal** is false, then you should also determine if one of the sets is a subset of the other set. If so, you should state an appropriate theorem or proposition and prove it.
   - If you determine that a conjecture is to prove that a **function is a bijection** is false, then you should determine if the function is an injection or is a surjection. You should then state an appropriate theorem and prove it.
2. See the **Guidelines for the Portfolio Project** for the important due dates and other rules for the Portfolio Project.
3. **Honor System**. All work that you submit for the Portfolio Project must be your own work. This means that you may not discuss the portfolio project with anyone except the instructor of the course and may not use any resources other than the textbook.
4. **Electronic Submission of Portfolio Problems**. Each solution or proof must be typeset with LaTeX. A Sample template file can be found on Blackboard.

   Each solution or proof for a portfolio problem must be submitted to the instructor electronically through the course's email MTH210FH@gmail.com. The instructor will make comments on the problem and return them to the student using the email. It is important to include the trackchanges.sty file in the same folder as your tex file in order for the template to compile.
5. **Deadlines and Due Dates**.
   (a) The deadline for office discussion and submission of Problems from Group 1 for review is Wed 1/30/2013.
   (b) The last day to submit Problems from Group 1 for a grade is Wed 2/6/2013.
   (c) The deadline for office discussion and submission of Problems from Group 2 for review is Wed 3/13/2013.
   (d) The last day to submit Problems from Group 2 for a grade is Wed 3/20/2013.
   (e) The deadline for office discussion and submission of Problems from Group 3 for review is Wed 4/10/2013.
   (f) The last day to submit Problems from Group 3 for a grade is Wed 4/17/2013.

Group #1 Problems

There are four problems in this group. Your portfolio must have complete solutions for three (and only three) of these problems. You may submit any problem for review at most two times but you may have at most seven submissions for review. Your third submission for a given problem will be considered the final submission and does not count as one of the seven submissions for review.

Portfolio Problem #1

Conjecture
For each integer m, $6m^2 - 12m + 5$ is an odd integer.

Portfolio Problem #2

Write a proof of the **Pythagorean Theorem**. To do this, on the Internet, go to the URL

`http://www.cut-the-knot.org/pythagoras/index.shtml`

and explore the proofs of the Pythagorean Theorem there. Choose one that you will learn and understand and write about for your portfolio.

This is to be your *only* Internet source for this paper.

While your proof should be modeled after one of the proofs presented at the above internet source, **this proof must be written according to the standards for MTH 210 and must be entirely in your own words and include all of the key steps required for a rigorous argument**.
Note: The proofs presented in your source often leave out key steps or details. Be sure you provide *full* details in your work so that every step is accounted for.

Portfolio Problem #3

For the following propositions, it might be helpful to answer the following questions. These answers do not have to be a part of your written work, and you can use any reference to recall the answers.

- What is an isosceles triangle?
- What is the Pythagorean theorem for right triangles?
- What is the formula for the area of a triangle?How does this formula apply to right triangles?

Prove the following propositions:

Proposition 1. For a right triangle, suppose that the hypotenuse has length c feet and the lengths of the sides are a feet and b feet. If the triangle is an isosceles triangle, then the area of the right triangle is $\frac{1}{4}c^2$.

Proposition 2. For a right triangle, suppose that the hypotenuse has length c feet. If the area of the right triangle is $\frac{1}{4}c^2$, then the triangle is an isosceles triangle.

Portfolio Problem #4

Conjecture.
The sum of the cubes of any three consecutive natural numbers is divisible by 3.

Group #2 Problems

There are five problems in this group. Your portfolio must have complete solutions for three (and only three) of these problems. In addition, your portfolio must contain Problem #8 or Problem #9 (or both). You may submit any problem for review at most two times but you may have at most seven submissions for review. Your third submission for a given problem will be considered the final submission and does not count as one of the seven submissions for review.

Portfolio Problem #5

Conjecture
For all natural numbers m and n, if m and n are both even, then $\left(m^2 + n^2\right)$ is not a perfect square.

Portfolio Problem #6

Conjecture 1
For each three digit natural number n, if a is the hundreds digit, b is the tens digit, and c is the units digit, then n is congruent to $(a + b + c)$ modulo 9.

Conjecture 2
For each three digit natural number n, if a is the hundreds digit, b is the tens digit, and c is the units digit, 9 divides n if and only if 9 divides $(a + b + c)$.

Suggestions: Try several examples of three digit numbers. For each example, determine if it is congruent (modulo 9) to the the indicated value. For example, if $n = 948$, then $9 + 4 + 8 = 21$. Noticed that $948 - 3 = 945$ and that 9 divides 945 . Therefore, $948 \equiv 3 \pmod 9$.

When we write a three digit number such as 948, what does the 9 represent? What does the 4 represent? What does the 8 represent?

For a general three digit number, you may want to do something like let a be the hundreds digit, let b be the tens digit, and let c be the units digit.

Portfolio Problem #7

For driver's license numbers issued in New York prior to September of 1992, the three digits preceding the last two of a male with birth month m and birth date b are represented by $63m + 2b$. For females the digits are $63m + 2b + 1$. Determine the dates of birth and sex(es) corresponding to numbers 527 and 453. **Explain your work based on a famous theorem that you studied.**

Portfolio Problem #8

Prove each of the following propositions.

Proposition A: For each integer x, if $x \not\equiv 0 \pmod 5$, then $x^2 \equiv 1 \pmod 5$ or $x^2 \equiv 4 \pmod 5$.

Proposition B: For each integer a, if $5|a^2$, then 5 divides a.

Proposition C: The real number $\sqrt{5}$ is an irrational number.

Portfolio Problem #9

For this problem you may assume that we have already proved that $\sqrt{2}, \sqrt{3}$ are both irrational numbers. Prove the following Proposition.

Proposion. The real number $\sqrt{2} + \sqrt{3}$ is an irrational number.

Be careful! The sum of two irrational numbers is not necessarily an irrational number. For example, $5 + \sqrt{3}$ is irrational and $3 - \sqrt{3}$ is irrational, but

$$\left(5 + \sqrt{3}\right) + \left(3 - \sqrt{3}\right) = 8$$

Which is a rational number.

Hint: $\left(\sqrt{2} + \sqrt{3}\right) \cdot \left(\sqrt{2} - \sqrt{3}\right) = \ldots$

Group #3 Problems

There are four problems in this group. Your portfolio must have complete solutions for all four of these problems, but you do have a choice of doing either Part A or Part B in Problem #12 and Problem #13. You may submit any problem for review at most two times but you may have at most three submissions for review. Any problem submitted after you have had three problems reviewed will be considered a final submission.

Portfolio Problem #10

Let $y = \dfrac{1}{5x-2}$. Determine $\dfrac{dy}{dx}$, $\dfrac{d^2y}{dx^2}$, $\dfrac{d^3y}{dx^3}$, and $\dfrac{d^4y}{dx^4}$.

Let n be a natural number and let $y = \dfrac{1}{5x-2}$. Formulate a conjecture for a formula for $\dfrac{d^ny}{dx^n}$. Then use induction to prove your conjecture.
Hint: $\dfrac{d^{n+1}y}{dx^{n+1}} = \dfrac{d}{dx}\left(\dfrac{d^ny}{dx^n}\right)$

Portfolio Problem #11

Assume that $f_1, f_2, \ldots, f_n, \ldots$ are Fibonacci numbers. Prove that for each natural number n, $f_1^2 + f_2^2 + \ldots + f_n^2 = f_n \cdot f_{n+1}$.

Portfolio Problem #12

Complete either Part A or Part B (not both)

Part A

Conjecture.
Let V and W be sets in a universal set U. Prove that if $(V \cap W^c) \cup (W \cap V^c) = W$ then $V = \emptyset$.

Part B

Conjecture.
Let A, B, and C be subsets of a universal set U. If

(i) $A \cap B = A \cap C$; and (ii) $A^c \cap B = A^c \cap C$,

then $B = C$.

Portfolio Problem #13

Complete either Part A or Part B (not both)

Part A

Conjecture

The function $f : (\mathbb{R} - \{5\}) \to (\mathbb{R} - \{3\})$ by $f(x) = \dfrac{3x}{x-5}$ is a bijection.

State your results in the form of a proposition, which must be stated completely and correctly. The conclusion of your proposition should be something like the following: The function f is (or is not) and injection and the function f is (or is not) a surjection.

Part B

Define the function $F : \mathcal{M}_{2,2}(\mathbb{R}) \to \mathbb{R}$ by

$$F\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = a + d$$

Is the function F an injection? Is the function F a surjection?

State your results in the form of a proposition, which must be stated completely and correctly. The conclusion of your proposition should be something like the following: The function F is (or is not) and injection and the function F is (or is not) a surjection.

This test has 4 questions, for a total of 80 points.
Answer each of the following questions in the space provided. Write in complete sentences and explain your reasoning wherever appropriate.

Name : ______________________________

1. Let $U = \{-7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6\}$ be the universal set.
   Let $A = \left\{x \in U \middle| (x-1)^2 > 2\right\}$ and $B = \left\{x \in U \middle| |x| \leq 2\right\}$.

8 (a) List the elements of A and the elements of B.
$A =$

$B =$

7 (b) Draw a Venn diagram that visualizes the sets U, A, and B.

7 (c) List the elements of $A - B$.

7 (d) List the elements of $(A \cap B)^c$.

8 (e) List the elements of $\mathcal{P}(B - A)$.

10 2. Determine the last two digits in the decimal representation of 7^{707}. Find the first few powers of 7 and then use the theorems to get to the power you want.

15 3. Let A and B be subsets of some universal set U. Use Algebra on sets to prove the following equation.

$$(A \cup B) - C = (A - C) \cup (B - C)$$

Proof. We will prove the result by showing that the left hand side is equal to the right hand side. Consider

$$\begin{aligned} (A \cup B) - C &= \\ &= \\ &= \\ &= \\ &= \\ &= \\ &= (A - C) \cup (B - C) \end{aligned}$$

4. **Proposition:** For all real numbers x and y, $|xy| = |x| \cdot |y|$.

   Proof. Let x and y be real numbers. We will show that $|xy| = |x| \cdot |y|$ by considering the cases

8 (a) List the 4 cases that you need to consider to prove this proposition

-
-
-
-

10 (b) Pick any 2 cases and prove them. Fill the blanks on the first case of your choice, and use the same style to prove another case.

In the case where , we notice that $|xy| =$ and $|x| \cdot |y| =$. This shows that .

Finish another case now.

MTH 210-02 — Take Home Question — Nov 7^{th}, 2012

F.Hindeleh

This take home portion has 2 questions, for a total of 30 points.
This take home problem is due on **Wednesday 11/7/12 at Noon**. You do not need to type your proof, but your writing must follow the guidelines..
Show all your work in details.

Name : ____________________

15 1. **Proposition:** Let $f_1, f_2, \ldots, f_n, \ldots$ be the Fibonacci numbers. For all natural numbers n, f_{5n} is a multiple of 5.

Hint: Look at Proposition 4.13 in your book.

Proof.

15 2. Prove that for each **odd** natural number n with $n \geq 3$,

$$\left(1+\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\cdots\left(1+\frac{(-1)^n}{n}\right)=1$$

This test has 5 questions, for a total of 110 points.
Answer each of the following questions in the space provided. Write in complete sentences and explain your reasoning wherever appropriate.

Name : ______________________________

15 1. Let A be the set of all MTH210 students in all the sections in the Winter semester of 2013. Define a relation $\sim$ on A by

For all $x, y \in A, x \sim y$ if and only if x and y have the same initials.

For example John Smith $\sim$ Jamal Snider.

Explain why $\sim$ is an equivalence relation on A.

2. Let P and Q be the statements defined as follows:

   P: For every real number x, there exists an integer n such that $x \leq n^2$
   Q: $(\exists n \in \mathbb{Z})(\forall x \in \mathbb{R})(x \leq n^2)$

   5 (a) Write P in symbolic form.

   5 (b) Write Q in words without using any symbols like $\exists$ or $\forall$.

   5 (c) Write the negation of P either in symbolic form or words.

   5 (d) Are P, and Q logically equivalent? Why? or Why not?

3. Let $f : \mathbb{Z}_6 \to \mathbb{Z}_4$ be defined by $f(x) = 3x \pmod 4$.

7 (a) Draw an arrow diagram to represent the function f.

6 (b) Is the function f injective? Explain your answer carefully.

5 (c) Is the function f surjective? Explain your answer carefully.

5 (d) Does the inverse of f exists? i.e is $f^{-1} : \mathbb{Z}_4 \to \mathbb{Z}_6$ a function? Explain your answer.

4. Complete the following statements citing **the definitions** carefully.

5 (a) Let x be an integer. We say that $x \equiv 5 \pmod 7$ provided that

5 (b) We know that $19 \not\equiv 9 \pmod 7$ since

7 (c) Let A, B and C be subsets of some universal set U. We say that $B \subseteq (A \cap C)$ provided that

7 (d) Let $f : \mathbb{Z}_6 \times \mathbb{Z} \to \mathbb{Z}_4$ is said to be injective provided that

8 (e) Let $f : \mathbb{Z}_6 \times \mathbb{Z} \to \mathbb{Z}_5 \times \mathbb{Q}$ is said to be surjective provided that

5. Determine if the following statements are true or false. If true cite the name of the theorem that gives you this conclusion, and if false provide a counter example.

5 (a) Let a be integers, and n be a natural number. If r is the unique reminder of a when divided by n, then $a \equiv r \pmod n$.

5 (b) If a and b are odd integers, then $a \cdot b$ is a perfect square.

5 (c) If A and B be subsets of some universal set U, then $A \cap B \subseteq B$.

5 (d) If A and B be subsets of some universal set U, then $(A \cap B)^c = B^c \cup A^c$.

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This take home portion has 3 questions, for a total of 70 points.
This take home problem is due on **Wed 12/12 at 2:00**. You do not need to type your proof, but your writing must follow proof guidelines.

Name : ______________________________

15 1. Let A, B, and C be nonempty sets. Let $f : A \to B$ and $g : B \to C$. If $g \circ f : A \to C$ is a surjection, then $g : B \to C$ is a surjection.

20 2. For all $[a], [b] \in \mathbb{Z}_3$, if $[a]^2 + [b]^2 = [0]$ then $[a] = 0$ and $[b] = 0$.
Hint: Use the contrapositive and cases in proofs.

3. For $(a, b), (c, d) \in \mathbb{R} \times \mathbb{R}$, define the relation $\sim$ on $\mathbb{R} \times \mathbb{R}$ by

$$(a, b) \sim (c, d) \leftrightarrow a^2 + b^2 = c^2 + d^2$$

20 (a) Show that the relation $\sim$ is an equivalence relation on $\mathbb{R} \times \mathbb{R}$.

10 (b) Use set builder notation to express the equivalence class $\left[\left(\frac{\sqrt{3}}{2}, \frac{-1}{2}\right)\right]$.

5 (c) What is the geometric interpretation of the equivalence class $\left[\left(\frac{\sqrt{3}}{2}, \frac{-1}{2}\right)\right]$.

Let no one ignorant of geometry enter this door.

ENTRANCE TO PLATO'S ACADEMY

MATH 341-02
Euclidean Geometry
MWF 11:00 - 11:50 am A-2-165 MAK
Fall 2014

Instructor: Dr. Firas Hindeleh
Office: A-2-116 MAK
email: hindelef@gvsu.edu
phone: (616) 331-3739
twitter: @firashindeleh

Office Hours: TWF 10:00 - 10:50
and by appointment

Prerequisites: Math 210 (Communicating in Mathematics) and either Math 227 (Linear Algebra I) or Math 322 (Geometry for Elementary Teachers).

Note: Math 227 is HIGHLY recommended, and should at least be taken concurrently. Students with a grade of C or lower in Math 210 are at great risk in *any* course with 210 as a prerequisite. Students who have not passed Math 210 or at least one of 227 and 322 will be automatically dropped by the dean's office at the end of the first week of classes.

Text: None. See further discussion below.

Why are we not requiring a text? Several reasons. One is that there simply isn't a good text that is written with the style and focus of what we at GVSU want to do in Math 341. But another is that we want the course to be a genuine experience in discovery – working without a book is very helpful in this regard. Plus, there's the matter of the nature of mathematics, put well in this quote from Leon Henkin:

> *One of the big misapprehensions about mathematics that we perpetrate in our classrooms is that the teacher always seems to know the answer to any problem that is discussed. This gives students the idea that there is a book somewhere with all the right answers to all of the interesting questions, and that teachers know those answers. And if one could get hold of the book, one would have everything settled. That's so unlike the true nature of mathematics.*

Course Web Page: Most announcements, assignments, and other course details will be communicated through the course home page, which you access by going to the Blackboard server (`http://mybb.gvsu.edu/`). **Note:** A userid and password are required to log in to the course page: these are the same as what you use to access the campus network.

You will be expected to access the course home page frequently (at least once for each day class meets) and respond accordingly to announcements and assignments posted there. In addition, it will be essential to be a regular user of email in this course. Be sure your preferred email address has been appropriately updated in your Banner.

Course Content and Goals: We will begin with an effort to challenge our own implicit assumptions about geometry. From there we will move into the study of formal axiomatic systems, including finite geometries. With a focus on the notion of proof throughout the course, we will proceed to develop neutral geometry, and then study the role of the parallel postulate in Euclidean and (some) non-Euclidean geometry. Roughly halfway through the term, we'll spend several weeks studying beautiful and powerful results on triangles and circles. The course will conclude with a study of transformations and isometries.

Grading: Grades will be based on the following items . . .

Homework (20%): Approximately every other week, there will be a set of homework problems collected for a grade. The homework problems are one of the most important parts of the course, as it is here that you will build your personal understanding. On each homework assignment, you are permitted to collaborate with at most two classmate; if you choose to do so, your collaborator's name must appear on your paper below your own, one write up from the team is sufficient. In addition, you are permitted (indeed encouraged) to meet with me to discuss your progress on the homework exercises.

Each homework assignment will consist of 6 problems, given 10-14 days in advance of when the assignment is due. On the class day prior to the date the assignment is due, 3 problems will be announced as the ones that will be graded. Only those 3 will be marked, each on a scale of 10 points. I will glance at the other three problems and award a mark of 10 points for all three together, based solely on effort. Obviously you are responsible for understanding the content of all the problems (not just the graded ones), as these homework exercises are one of the key ways you prepare yourself for exams.

Besides myself and your group members, no other collaboration on graded homework exercises is allowed. Violating this policy will be grounds for a minimum penalty of zero on the assignment. See the syllabus addendum on the course policy for plagiarism for further details.

Lab and In-Class Activities (10%): On Fridays, we have the opportunity to use technology to explore the course content in a different way. While we may not use the computer every week, we will strive to use software that is useful for studying and discovering geometry as often as possible. Certain lab activities will be collected; such work will be completed in groups, where each student will receive the same grade. In addition, we will often engage in group activities in class, some of which may be marked for a group grade.

During those times when we are not using the computer, please keep the monitors stowed.

Semester Project (30%): Throughout the term you will work (with up to three peers) on writing a geometry journal that summarizes the work that we accomplish in class. Full details will follow in a separate document.

Exams (25%): There will be three one-hour examinations; it is possible that one or more of these may be accompanied by a small take-home portion. These tests are tentatively scheduled for the following dates: Monday September 22^{nd}, Friday October 24^{th}, and Monday November 24^{th}.

Final Exam (15%): The comprehensive final exam will be given Wednesday, December 10^{th}, 10:00-11:50 am at MAK A-2-165.

Course Grade:[1] Final grades will be no lower than as determined by the following scale: A 93%, A- 90%, B+ 87%, B 83%, B- 80%, C+ 77%, C 73%, C- 70%, D+, 67%, D 63%, F below 63%.

Expectations:

Attendance: Attendance is critical to your success, and will be taken during each class meeting. You are responsible for all announcements made in class concerning material covered, assignments, changes in the syllabus, or anything else pertinent to the course.

Academic Honesty: Some of the work in this course will be done collaboratively. It may be particularly helpful to meet once a week with other students to summarize and review key ideas in the course; indeed, this is essential to the semester project. Even on homework, some collaboration is permitted. However, please understand that there is a difference between collaboration and plagiarism. Collaboration requires you to contribute to solutions and think when you write. Copying any portion of someone else's work, no matter how small, is plagiarism! Evidence of plagiarism in an assignment will result in a minimum penalty of zero on the paper and possible action under the guidelines of the GVSU catalog. See complete details on plagiarism and academic honesty in the syllabus addendum on the subject.

You are expected to show integrity in all your work, and to encourage your peers to do likewise. I reserve the right to discuss the nature of your work with you prior to deciding a grade on an assignment.

Graded Work: On all graded assignments, I expect your very best. For homework problems to be handed in, please use pencil on loose-leaf paper, stapled if there is more than one page. Problems should be written in complete mathematical style and in your best handwriting. Graded papers will not be accepted late; papers that are due must be submitted at the beginning of the class on the due date.

Workload and Study Habits: For this 3-credit course, you are expected to spend at least 2-3 hours outside of class for each hour in class that we meet - average (and successful) students in previous semesters have reported an average of 9 hours of studying per 3 hour week of class (while some students report significantly more).

This time should be divided into two principal activities: developing a careful set of course notes as part of the semester textbook project, and solving problems. The former should be done (possibly in collaboration with your group) on a daily basis: take your class notes and summarize them succinctly to note key definitions, theorems, examples, and ideas. (The process of typesetting at least one of these days per week will essentially be your "textbook" project for the course.) All of this studying will be immensely helpful come exam time. The second type of regular work on solving problems will be in response to posted homework assignments and lab activities; these should be approached in an incremental way, attempting to do a little bit at a time, rather than all the work in the evening before the assignment is due.

Participation: In every class meeting, there will be significant opportunities for you to actively

[1] Note well that overall percentage grades are generally **not** rounded up: an average of 82.8%, for example, would earn a grade of B-.

participate aloud. Through questions asked in lecture and brief exercises for teams of 2-3, you will be able to check and demonstrate your understanding in class. You must come prepared to class in order to effectively participate. I expect that everyone will share in this important aspect of our learning process.

Drop Date: The drop deadline for this course is Friday, October 24, at 5 pm.

Holidays: There will be no classes Monday September 1 due to labor day, and from Nov 26-28 due to Thanksgiving Break.

Student Concerns: If there is any student in this class who has special needs because of learning, physical or other disability, please contact me or the Disability Support Resources office (200 STU, 616-331-2490). Furthermore, if you have a disability and think you will need assistance evacuating this classroom and/or building in an emergency situation, please make me aware so GVSU can develop a plan to assist you.

Feedback: I am always willing to discuss with you any thoughts you have about the course, including both your performance and mine. Please let me know if you think something could be better or if you like something that we are doing.

On the potential impact of geometry:

> *At the age of 12 I experienced a second wonder of a totally different nature: in a little book dealing with Euclidean plane geometry, which came into my hands at the beginning of a school year. Here were assertions, as for example the intersection of the three altitudes of a triangle in one point, which - though by no means evident - could nevertheless be proved with such certainty that any doubt appeared to be out of the question. This lucidity and certainty made an indescribable impression upon me. For example I remember that an uncle told me the Pythagorean theorem before the holy geometry booklet had come into my hands. After much effort I succeeded in "proving" this theorem on the basis of the similarity of triangles ... for anyone who experiences [these feelings] for the first time, it is marvelous enough that man is capable at all to reach such a degree of certainty and purity in pure thinking as the Greeks showed us for the first time to be possible in geometry.*[2]

[2] From pp. 9-11 in the opening autobiographical sketch of *Albert Einstein: Philosopher-Scientist*, edited by Paul Arthur Schilpp, published in 1951.

MTH 341-02
Fall 2014
Tentative Daily Schedule

F. Hindeleh

		Monday	Tuesday	Wednesday	Thursday	Friday
Aug 25-29	Wk 1	(01) intro_to_finite_geometries.tex (4 point)		(02) four line geometry; Fano's geometry		(03) Lab 1 on Spherical Geom: due Thursday
Sep 1-5	Wk 2	Labor Day		15 more minutes on S geometry (04) Introduction to SMSG axioms		(05) Lab 2 on hyperbolic geometry **Lab 1 Due**
Sep 8-12	Wk 3	(06) NeutralTheorems1 (two proofs)		(08) Neutral Theorems 1a (Exterior Angle Theorem); **Hwk 1 Due**		(07) Lab 3 on GSP
Sep 15-19	Wk 4	(08b) Building Triangles; **Text Check 1**		(09) Neutral Theorems 2 (ASA, etc.)		(10) Neutral Theorems 3 Inverse of IST **Lab 3 Due**
Sep 22-26	Wk 5	**Test 1**		**open day (10 continued) Inverse of IST, Exercise on Right Triangles**		(11) The Role of Parallels - Lab 4 in hyperbolic space
Sep 29-Oct 3	Wk 6	AIAT Revisited; History of the Parallel Postulate **Hwk 2 Due**		(12) Neutral Theorems 4 (AIAT and corollaries)		(13) Lab 5 - Towards Euclid
Oct 6-10	Wk 7	(14) -- Parallelograms and Quadrilaterals intro		(15) rhombi and kites		more 15; start 16 **Lab 5 due**
Oct 13-17	Wk 8	(16) quadrilateral tree **Hwk 3 Due**		(17a) - Just handout; (18) 3PT, nPT, MTT		MTT through median concurrence (E7-E9) (20) median concurrency
Oct 20-24	Wk 9	(19) area **Text Check 2**		area summary;		**Test 2**
Oct 27-31	Wk 10	(21a) basic proportionality converse+/similar triangles; picking up remaining pieces		diagonal definitions for trapezoids and isosceles trapezoids	(21) altitude conc (hwk)	(22) Euler Line - lab
Nov 3-7	Wk 11	(23) 9 point circle **Hwk 4 Due**		coordinates, lines, similar triangles, and foundational ideas		Pythagoras and trigonometry **Euler line lab due**
Nov 10-14	Wk 12	**(24)** isometry intro/fxn terminology		the land of linear algebra start (26) Exploring Isometries		(26) exploring isometries **Hwk 5 Due**
Nov 17-21	Wk 13	Matrices of isometries **Text Check 3**		transformation exercises (27)		(28) Reflections Galore
Nov 24-28	Wk 14	**Test 3**		Thanksgiving		Thanksgiving
Dec 1-5	Wk 15	(29) Theorems on Euclidean Motions **Hwk 6 Due**		More matrices of isometries		(30) The Power of Reflections
Dec 8-12	Finals			**Final 10:00- 11:50 AM MAK A-2-165**		

Hindeleh

Math 341
Homework 4
due Monday, November 7

Directions: Complete all of the following exercises in accordance with standard expectations for writing and mathematical style from Math 210. In each case, be certain to fully demonstrate your reasoning and solution to each problem. You may either typeset your work or neatly handwrite your solutions in pencil on loose-leaf paper. Clearly state the problem you are solving in each case, as appropriate.

You are permitted to collaborate with your group; and turn in one assignment per group. The only other sources you are allowed to use are your course notes and help from the instructor.

Three of the following six questions will each be graded in detail and marked on a scale of 10 points each; the remaining three problems will together receive one 10 point grade based largely on effort and completeness in addressing the big ideas. The three problems that will be graded will be announced on Friday, November 4. Please note that no assistance from me on this assignment will be available after 3 pm on the 4th.

1. Let convex quadrilateral $ABCD$ be given (note that the quadrilateral is arbitrary – nothing is assumed about sides, angles, etc.) Construct the midpoint of each of the four sides of $ABCD$, and use these four new points W, X, Y, Z to construct a new quadrilateral. What can you say about quadrilateral $WXYZ$? State your observation as a theorem and prove it.

2. Let $\triangle ABC$ be given. Construct the midpoint of each of the three sides, say P, Q, and R, and use these to build a new triangle (the so-called *medial* triangle). What can you say about the four small triangles that result? State your observation as a theorem and prove it.

3. To date, we have proved that the perpendicular bisectors of any Euclidean triangle are concurrent (at the *circumcenter*), the angle bisectors of any triangle are concurrent (at the *incenter*), and the medians of any Euclidean triangle are concurrent (at the *centroid*). An **altitude** of a triangle is a line which passes through a vertex and is perpendicular to the opposite side.

   In this problem, you will prove the following result: **Theorem.** *The altitudes of any Euclidean triangle are concurrent (at a point called the* orthocenter*).*

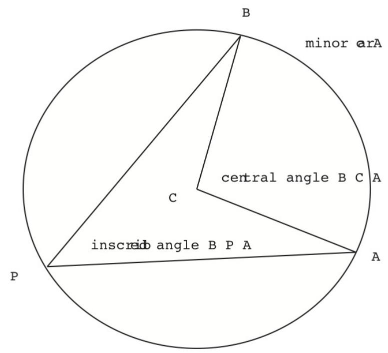
   Here is a hint to help you get started proving that the altitudes of a triangle are concurrent. (For your proof, you may assume that the given triangle is acute; do note, however, that the proof must be appropriately modified for the cases the triangle is right or obtuse.)

   One idea for the proof is to build a new triangle $\triangle XYZ$ from $\triangle ABC$, the one you are given. Do this by constructing through each vertex of $\triangle ABC$ the line parallel to the opposite side of the triangle. Call the points where these three new lines meet X, Y, and Z. (Side note: why *must* these new lines intersect?)

   What do you notice about the altitudes of $\triangle ABC$ when considered from the point of view of the triangle $\triangle XYZ$? Use this to construct a proof of concurrency. Include a labeled plot from Sketchpad in your homework solutions. Our work with parallelograms will play a key role in your argument.

4. Next, we begin to explore some interesting properties of circles, as well as how triangles and circles are related to one another. Recall that, by definition, a circle is *the set of all points that are equidistant from a fixed point, C. C* is called the *center* of the circle, and the distance from C to any point on the circle is the *radius* of C.

   Definition. Any angle whose vertex is the center of a given circle is called a *central angle* of the circle (for example, $\angle ACB$ in the figure above). An angle $\angle APB$ whose vertex P is on the circle itself (along with A and B) is called an *inscribed angle*.



Various measures of central angles produce *semicircles*, *minor arcs* (less than half a circle), and *major arcs* (more than half a circle), and we measure the arcs on the circle in degrees by using the measure of the central angle.

Here are two questions related to central and inscribed angles. In Geometer's Sketchpad, construct a circle with center C and diameter $\overline{PB}$. Construct a point A on the circle, and consider inscribed angle APB. Label the center of the circle C and construct central angle ACB. Say that $m\angle ACB = x°$.

- What can you say about the measure of $\angle APB$? State your conjecture as a theorem and prove it.
- In the same situation as above, what can you say about the measure of $\angle PAB$? (Think about the two smaller angles that make up $\angle PAB$ and what their measures are in terms of x.) Again, provide a proof of your observation.

5. Recall your observation in the previous problem about how the measure of inscribed angle APB (when $\overline{PB}$ is a diameter of the circle) is related to the measure of central angle ACB. **Is this true even if $\overline{PB}$ is not a diameter of the circle?**

   Note that there are two cases to consider: where the center C is interior to $\angle APB$ and exterior to $\angle APB$.

   Be sure to construct a figure in GSP where you can simultaneously measure an inscribed angle and the corresponding central angle. Then make an appropriate conjecture by filling in the blank below, and prove the result.

   Theorem. If an angle is inscribed in a circle, then the angle's measure is ____________________.

6. State and prove the Pythagorean Theorem (be sure that you provide a careful and precise statement of the theorem). To do so, on the Internet, go to the URL

   `http://www.cut-the-knot.org/pythagoras/index.shtml`

   and explore the proofs of the Pythagorean Theorem there. Choose one that you will learn, understand, and present in your homework. Note well: the proofs presented at this URL are missing key details in the construction of figures and more. You need to fill in the details and provide a rigorous argument to prove the theorem. (For example, if a proof uses a square divided up into pieces, you cannot assume that the square is given to you as pictured; you have to explain how to construct it *in terms of* the given right triangle provided by the hypothesis of the Pythagorean Theorem.) You should choose a proof of the result that does **not** use similar triangles.

Directions: Pick one of the problems below, rewrite it in modern language.

Nasir al-Dīn al-Tūsī's *The Complete Quadrilateral* (c. 1250)

From Book III, Chapter 3:

If two different arcs of a circle that, together, are less than a semicircle, join at a point, and their sum is known, and if the ratio of the Sine of one of them to the Sine of the other is known, then each of them is known.

Let there be in circle *ABG* two arcs, *AB* and *AG*, which join at *A*, and let their sum, *BAG*, which is less than a semicircle, be known. Let the ratio of the sine of arc *AB* to the sine of arc *AG* be known. I say, then, that each of the two arcs, *AB* and *AG*, is known (Figure 9).

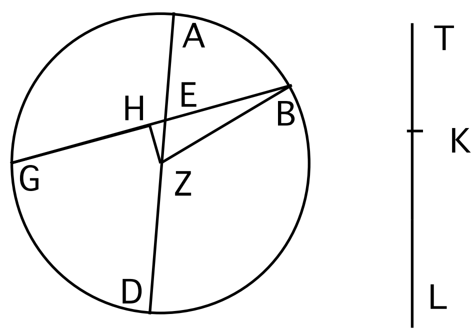


Figure 9

Its Proof: The chord *BG* and diameter *AD* are produced, and so they meet at [say] *E*. From the center, *Z*, the perpendicular *ZH* is produced onto *BG*, and *BZ* is joined. Because arc *BAG* is known, the chord *BG* is known. And because Sin(*BA*)/Sin(*AG*) is known *BE*/*EG* is known. So, let it be as the ratio of *TK* to *KL*, and *BG*/*BE* = *TL*/*TK*, so *BE* is known and [hence] *EG* is known. And *BH*, half of *BG*, is known, and so *EH* is known. And *ZH*, the Sine of the complement of half the arc *BG* is known. And in the right triangle *EZH* the two sides *EH* and *HZ* containing the right angle are known, so angle *EZH* is known. And angle *BZH*, which is in magnitude half of arc *BAG*, is known. So the remaining angle, *BZA*, is known, and it is the magnitude of arc *BA*, [which is therefore] known. And [so] the remaining arc *AG* is known as well. And that is what we wanted to demonstrate.

QUESTION:

1. Solve this problem in modern terms: Given the sum of two angles and ratio of their sines, determine the two angles.
2. Should this problem be in trigonometry texts? (I do not recall ever seeing

Abū Kāmil's *Constructing an Equilateral Pentagon in a Given Square* (c. 900)

And if it is said to you: A quadrilateral *ABGD* is given, whose sides are equal, each of whose angles is right, and each of whose sides is equal to 10, in which we shall inscribe an [equilateral] pentagon in this [way].

Let it be the pentagon *AEHZM* (Figure 3). And so the knowledge of each side of the pentagon is that you make one of the sides of the pentagon, say the line, *AE*, a thing. Then remains the line *BE*, ten less a thing. And the line *GH* will be the root of one-half mal. So there remains the line *HB* [equal to] ten less the root of one-half mal (square). And the line *EB* is ten less a thing. And so we multiply each of the two by its like and we put the two of them [the resulting squares] together and it will be two-hundred dirhams and mal and half mal less twenty things and less the root of two-hundred mal equal to mal. Solve this along the lines I have shown you and *AE*, a side of the pentagon, is the root of the sum of 200 and the root of 320,000 subtracted from 20 and the root of 200.

QUESTION:

1. Check Abū Kāmil's solution to this problem. Note that this pentagon is not regular.
2. Can one use this problem in a geometry class, assuming the students have studied quadratic equations? Or in a class for prospective teachers of geometry?

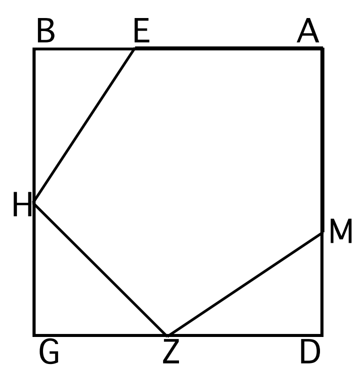


Figure 3

MTH 341-02 — Test 2 — Oct 24^{th}, 2014 — **F. Hindeleh**

This test has 3 questions, for a total of 100 points.
Answer the questions in the spaces provided on the question sheets. **Show all your work in details**.

Name : ______________________________

1. For each of the following statements, decide whether the statement is TRUE or FALSE. In either case, you must provide one or two sentences of justification in support of your choice.

   10 (a) In Neutral Geometry, if two lines l and m are met by a transversal t, the alternate interior angles created are congruent if and only if the lines l and m are parallel.

   10 (b) In Hyperbolic Geometry, the angle bisectors of any triangle ABC are concurrent at a point that is equidistant from each of the three sides of the triangle.

   10 (c) In Neutral Geometry, if lines l and m are parallel, and line t is perpendicular to line l, then t is also perpendicular to line m.

   10 (d) In Euclidean Geometry, the measure of any exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles.

2. In each of the following questions you should give a short answer response by writing 2-3 sentences. Be focused and to the point; let the space provided guide you in how much is appropriate to write. If a particular theorem is relevant, cite it.

10 (a) Given a quadrilateral $ABCD$ that is a square, what other valid names may $ABCD$ be called? Answer from among "parallelogram", "kite", "rectangle", "trapezoid", "isosceles trapezoid", and "rhombus." Justify **one** of your claims.

10 (b) A middle school student studying quadrilaterals suggests the following definition for "rectangle": "a parallelogram with at least one right angle." Is her definition a valid suggestion? Explain using things we've proved about quadrilaterals.

10 (c) If $\triangle XYZ$ is a right triangle (with the right angle at Z) and M is the midpoint of the hypotenuse $\overline{XY}$, explain why $\overline{MZ}$ divides $\triangle XYZ$ into two isosceles triangles.

10 (d) Given $\triangle ABC$ with the points X, Y, Z are midpoints of $\overline{BC}, \overline{AC}, \overline{AB}$ respectively. Explain why $\triangle YCX \cong \triangle ZXB$.

20 3. Prove **Theorem Q2** (Rectangles): A quadrilateral has four right angles if and only if its diagonals are congruent and bisect each other.

This take home exam has 2 question, for a total of 60 points.
Answer the question in the spaces provided on the question sheets showing all your work in detail. This is due on Wed 12/11/13 at 10AM

Name : ____________________

1. For each of the following parts, include a Geometer's Sketchpad graph of the nine-point circle of the triangle ΔABC. How many distinct points related to the triangle does the nine point circle contain if

10 (a) ΔABC is isosceles.

10 (b) ΔABC is equilateral.

In each part discuss why certain points coincide.

2. Consider the **Geometry of Pappus**:

Undefined terms: point, line point on a line.

Axioms:

I. There exists at least one line.

II. Every line has exactly three points.

III. Not all points are on the same line.

IV. If a point P is not on a given line l, then there exists exactly one line m that contains P and isparallel to l.

V. If a point P is not on a given line l, then there exists exactly one point P' on l such that no line contains both P and P'.

VI. With the exception of Axiom V, if P and Q are distinct points, then exactly one line contains them both.

20 (a) Draw a diagram that models this geometry. Explain your construction step by step.

20 (b) Assuming that this geometry has 9 points, how many lines are in this geometry? Write down your conjecture in the form of a proposition, then prove it.

MTH 341-02 Dec 11^{th}, 2013
Fall 2013 FINAL TAKE HOME **F. Hindeleh**

This take home exam has 3 question, for a total of 140 points.
Answer the question in the spaces provided on the question sheets showing all your work in detail.

Name : ______________________________

1. For each of the following statements, decide whether the statement is TRUE or FALSE. In either case, you must provide one or two sentences of justification in support of your choice.

15 (a) The circumcenter of an equilateral triangle is the same as its orthocenter.

10 (b) In Spherical Geometry, the side-angle-side criterion for congruent triangle is valid.

15 (c) The diagonals of the isosceles trapezoid are congruent.

10 (d) The composition of two parallel reflections about a vertical axis is a reflection about another vertical axis. i.e for all $a, b \in \mathbb{R}$ there exists a $c \in \mathbb{R}$ such that $R_{x=a} \circ R_{x=b} = R_{x=c}$.

15 (e) The composition of two rotations centered at the origin is a rotations centered at the origin. i.e for all $\alpha, \beta \in \mathbb{R}$ there exists a $\gamma \in \mathbb{R}$ such that $R_{(0,0),\alpha} \circ R_{(0,0),\beta} = R_{(0,0),\gamma}$.

15 (f) For ΔABC, if H, S, and M are the orthocenter, circumcenter, and the centroid respectively, then $HS = 2SM$.

15 (g) The function $f : \mathbb{E}^2 \to \mathbb{E}^2$ associated with the matrix $A = \begin{bmatrix} 1 & -4 \\ 2 & -8 \end{bmatrix}$ is an isometry.

i.e $f(x, y) = \begin{bmatrix} 1 & -4 \\ 2 & -8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$.

15 (h) For any right ΔABC, the center of the circle that contains all three vertices (circumscribe circle) is the midpoint of the hypotenuse.

15 2. Find the image of the triangle with vertices $(0,0), (-1,0), (1,-1)$ reflecting it through the line $y = -\frac{3}{4}x$ **then** by translating it via the vector $\mathbf{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$. Use exact numbers and not calculator approximations.

Hint: $\sin(2\theta) = 2\sin\theta \cdot \cos\theta$ and $\cos(2\theta) = \cos^2\theta - \sin^2\theta$

15 3. Find the matrix of the inverse transformation of the isometry in the previous question.

Mathematics 408
Advanced Calculus I

Section 01
Winter 2014

TR 10:00 - 11:15 A.M. MAK B-1-122

Instructor: Dr. Firas Hindeleh
MAK A-2-116
(616) 331-3739
hindelef@gvsu.edu
Office Hours: TWR 2:00 - 2:50 P.M.,
or by appointment

Prereqisites: Math 203 and MTH 210 with a grade of C or better.

Text: *Math 408 Notes and Exercises Course-Pack*, Winter 2011 edition, by A. Tefera.

Course Content: Key topics to be studied include *the real number system, sequences, functions limits, continuity, uniform continuity, differentiation and Riemann integration.*

Course Goals: This course introduces students to the fundamental concepts and techniques of real analysis. This course aims to enable students to develop critical thinking ability and creativity in solving problems and proving theorems. An ability to read and write proofs will be stressed. Upon completion of this course, students will be able to:

- State, explain and apply the completeness, the Archimedean and density properties of the set of real numbers.
- Define, explain and calculate infs and sups.
- Comprehend the various definitions related to sequences and use them to prove and derive related results.
- Comprehend the various definitions related to limits of functions and use them to prove and derive related results.
- Define and determine continuity or uniform continuity of a function on a given set.
- Define and determine differentiability of a function.
- Apply the Mean Value Theorem, Rolles Theorem and related results to solve a variety of problems.
- Define and determine the Riemann integrablity of a function on a given interval.

Grading: Your final grade will be determined in the following way:

Homework	25%
Portfolio	30%
Exams	30%
Final Exam	15%

The final grade assigned to you will not be lower than that computed in this way. However, I reserve the right to raise your grade if you are an active, thoughtful participant in class and there is evidence you are working consistently and with care.

Homework: You will receive homework assignments every week through Bb. Homework is due at the beginning of Tuesdays class. You are encouraged to work on your homework exercises in groups of two in which case one write up per group is sufficient. The importance of working through all the problems carefully cannot be overstated: working problems is the absolute best way to learn mathematics. Homework should be written according to proof writing guidelines you learned in MTH 210. See guidelines for proof writing document.

Portfolio Problems: You will turn in nine mathematical problems from 3 group of problems that will be included in a "MTH 408 Portfolio". You may turn in each of these problems one time to be critiqued. After each submission, you have the opportunity to rewrite your portfolio proof prior to submission for a final grade. However, no more than one problem may be submitted for review on a given day, and no more than four problems (or essay) may be submitted for review during any week. Do not delay working on those problem till before the due date.

The portfolio will be graded on a 100 point scale. Each problem will be worth 10 points (for a total of 100 points), and in addition, there will be 10 points possible for submission of drafts for review by the professor before the due date. Information about this is included in the "Guidelines for the Portfolio Project" document.

The portfolio problems must be completed using LaTeX. You can download the free version of LaTeXfrom the web. If you prefer a web-based version of LaTeX, you may setup a free account with `www.writelatex.com` and link it with your Dropbox. More details on the process of writing and rewriting as well as due dates for submissions of drafts are in the "*Guidelines for Portfolio Project*" file.

The last date to submit a portfolio problem for review is Monday April 7, 2014, and the last day to submit a portfolio problem is Monday April 14, 2014. More details on the process of writing and rewriting as well as due dates for submissions of drafts will be given within the first week of the course.

Exams: There will be two midterms scheduled for Tuesdays Feb 4, and Mar 18. There might be a take home portion that will be due on the same exam day.

Note well: No makeup exams will be given. If you are unable to attend an exam, it is your responsibility to notify me *prior* to the exam so that suitable arrangements may be made.

Final Exam: The final exam will be Wednesday, April 23 from 8:00 - 9:50 A.M in MAK B-1-122.

Final grade: Shown below is the percentage required to obtain a particular grade:

Grade	D	D-	C-	C	C+	B-	B	B+	A-	A
Percentage	60.0	67.0	70.0	73.0	77.0	80.0	83.0	87.0	90.0	93.0

Drop Date: The deadline for withdrawing from this course is Friday, March 7 at 5 P.M.

Expectations:

Attendance: Attendance is expected and critical to your success. Please be on time to class. You are responsible for all announcements made in class concerning material covered, assignments or anything else relevant to the course.

Academic Honesty: Some of the work in this course will be done collaboratively. It is also recommended that you form a study group to assist in learning the material and completing daily homework problems. However, it is important for you to understand that there is a difference between collaboration and plagiarism. Collaboration requires you to contribute to solutions and to think when you write. Representing someone else's work, no matter how small, as your own is plagiarism, a serious offense that will be met with a grade of zero and possible action under GVSU's guidelines. You are expected to show integrity in all your work and to encourage the same in your colleagues.

Mathematical Communication: One important part of mathematics is its emphasis on the clear and careful presentation of reasoning. This includes clearly stating the problem, making important observations in complete sentences, writing additional thoughts to clarify symbolic expressions, and showing a clear overall progression from problem to solution. The quality of your presentation and writing will count in all of your graded work.

Work load: To be successful in this course, you will need to work hard and consistently. A good rule of thumb is that you should spend at least two hours working outside of class for every hour of class time. Besides working on homework and other assignments, you should also keep a well-organized record of all your study notes and completed problems for future reference. In spite of the fact that we will discuss the most important concepts in detail during class, you will be expected to learn and assimilate many other ideas independently. Please take advantage of my office hours to discuss with me any problems you are having.

Participation: In every class meeting, you will be expected to participate actively and to share your understanding with the class. To do so effectively, you must come to class prepared. I expect that everyone will share in this important aspect of our learning process.

Late work: Late work will not be accepted. Any assignment is due on the stated date at the beginning of class.

Feedback: I am always happy to discuss with you any thoughts you have about the course including both your performance and mine. Please let me know if you think something could be better or if you like something that we are doing.

Student Concerns: Any student who requires accommodation because of a physical or learning disability must contact Disability Support Resources (`http://www.gvsu.edu/dsr`) at (616) 331-2490 as soon as possible. After you have documented your disability, please make an appointment or see me during office hours to discuss your specific needs.

Any student needing academic accommodations beyond those given to the entire class please be advised that the University's Office of Disability Support Services (DSR, ext. 12490) is available to all GVSU students. It is the student's responsibility to request assistance from DSS.

Directions

1. See the **Guidelines for the Portfolio Project** for the important due dates and other rules for the Portfolio Project.

2. **Honor System**. All work that you submit for the Portfolio Project must be your own work. This means that you may not discuss the portfolio project with anyone except the instructor of the course and may not use any resources other than the textbook.

3. **Electronic Submission of Portfolio Problems**. Each solution or proof must be done on LaTeX. A template can be found on Bb for convenience.

   Please turn in both the tex and pdf files electronically through the course's email

   MTH408FH@gmail.com. The instructor will make comments on the problem and return them to the student using the email.

4. **Deadlines and Due Dates**.

   (a) The deadline for office discussion and submission of Problems from Group 1 for review is Monday 1/27/14.

   (b) The last day to submit Problems from Group 1 for a grade is Monday 2/3/14.

   (c) The deadline for office discussion and submission of Problems from Group 2 for review is Monday 3/10/14.

   (d) The last day to submit Problems from Group 2 for a grade is Monday 3/17/14.

   (e) The deadline for office discussion and submission of Problems from Group 3 for review is Monday 4/7/14.

   (f) The last day to submit Problems from Group 3 for a grade is Monday 4/14/14.

Group #1 Problems

There are four problems in this group. Your portfolio must have complete solutions for three (and only three) of these problems. You may submit any problem once for review. Your second submission for a given problem will be considered the final submission and will be awarded a grade.

Portfolio Problem #1

Conjecture
Let $c \in \mathbb{R}$ such that $c > 1$. Show that for all $n \in \mathbb{N}$, $c^n \geq c$ with equality occurring if and only if $n = 1$.

Portfolio Problem #2

Let $x, y, a, b \in \mathbb{R}$. Suppose that there exists an $\epsilon > 0$ such that $x \in V_\epsilon(a)$ and $y \in V_\epsilon(b)$. Prove that

$$|xy - ab| < (|a| + |b|)\epsilon + \epsilon^2$$

Portfolio Problem #3

Let S be a nonempty bounded subset of $\mathbb{R}$. Let $a \in \mathbb{R}$ such that $a < 0$. Define aS by $aS = \{as | s \in S\}$. Prove that inf(aS)=a sup(S) and that sup(aS)=a inf(S).

Portfolio Problem #4

If $x > 0$, then there exists an $n \in \mathbb{N}$ such that $\dfrac{1}{3^n} < x$.

Group #2 Problems

There are four problems in this group. Your portfolio must have complete solutions for three (and only three) of these problems. You may submit any problem once for review. Your second submission for a given problem will be considered the final submission and will be awarded a grade.

Portfolio Problem #5

Using the $\epsilon - n_0$ definition of the limit of a sequence, prove that if $\lim(x_n) = L$, where $L \neq 0$, then there exists an $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$, $|x_n| > \dfrac{|L|}{2}$.

Portfolio Problem #6

Using the definition of Cauchy Sequences, prove that if (x_n) and (y_n) are Cauchy then $(x_n + y_n)$ is Cauchy.

Portfolio Problem #7

Let (x_n) and (y_n) be sequences of real numbers and suppose that $\lim\left(\dfrac{x_n}{y_n}\right) = \infty$.

- Show that if $\lim(y_n) = \infty$, then $\lim(x_n) = \infty$.
- Show that if (x_n) is bounded, then $\lim(y_n) = 0$.

Portfolio Problem #8

Let $x_1 = 1$ and for $n \geq 2$, $x_n = \sqrt{3 + x_{n-1}}$. Show that x_n converges and find the limit.

Group #3 Problems

There are three problems in this group. Your portfolio must have complete solutions for all three of these problems. You may submit any problem once for review. Your second submission for a given problem will be considered the final submission and will be awarded a grade.

Portfolio Problem #9

Suppose that for all $n \in \mathbb{N}$, $x_n \geq 0$, and that $\lim\left((-1)^n x_n\right)$ exists. Show that (x_n) converges.
Hint: First show that if $\lim\left((-1)^n x_n\right)$ exists, then $\lim\left((-1)^n x_n\right) = 0$.

Portfolio Problem #10

Use the $\epsilon - \delta$ definition to show that $\displaystyle\lim_{x\to -1} \frac{2x+10}{4x+5} = 8$.

Portfolio Problem #11

For all $x \in \mathbb{R}$ let $f, g : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x + 2$ and

$$g(x) = \begin{cases} -2 & x \neq 4 \\ 0 & x = 4 \end{cases}$$

Show that $\displaystyle\lim_{x\to 2}(g \circ f)(x) \neq (g \circ f)(2)$. Why doesn't this contradict Theorem C.2.2?

This Homework has 3 questions, for a total of 40 points.
Turn one writeup per group on Thu Jan 30. **Proofs must follow writing guidelines from MTH 210**.

Name(s) : ______________________

1. Given that $S = \left\{1 - \frac{1}{5^n} : n \in \mathbb{N}\right\}$.

[5] (a) Is S bounded? Justify your answer. [Hint: Show that for all $s \in S, |s| \leq M$.]

[7] (b) Do sup S and inf S exist? If so, determine them and prove your answers.

[10] 2. Let A and B be non-empty subsets of $\mathbb{R}$. Prove that if $A \subset B$ then

$$\inf B \leq \inf A \leq \sup A \leq \sup B.$$

[8] 3. (a) Prove that if x is an upper bound of a set $S \subset \mathbb{R}$ and $x \in S$, then $x = \sup S$.

[5] (b) Make and prove an analogous statement for the infimum of S.

[5] (c) Show by a counterexample that the converse of each statement in (a) and (b) is false.

Oct 15^{th}, 2013

MTH 408-01 HOMEWORK 5 **F. Hindeleh**

This Homework has 3 questions, for a total of 40 points.
Turn one writeup per group on Thursday Oct 24. **Proofs must follow writing guidelines from MTH 210**.

Name(s) : ______________________________

[10] 1. Let (x_n) be a sequence defined as follows:
$x_1 = \sup\{\sin 1, \sin 2, \sin 3, \ldots\}$, $x_2 = \sup\{\sin 2, \sin 3, \sin 4, \ldots\}$, $x_3 = \sup\{\sin 3, \sin 4, \sin 5, \ldots\}\ldots.$
Does (x_n) converge? Justify.

[15] 2. Define a sequence by $x_1 = a$ and for all $n \geq 2$, $x_{n+1} = \sqrt{a + x_n}$. For what values of a ($a > 0$) will this sequence converge? and what would the limit be in this case? Prove your conjecture.

[15] 3. Define a sequence by $x_1 = 3$ and for all $n \geq 2$, $x_n = \dfrac{1}{4 - x_{n-1}}$. Show that (x_n) converges and find the limit.

This Test has 1 questions, for a total of 40 points.
Show all your work in details.

Name : ______________________________

1. Evaluate each of the following limits (if they exist) or prove that the limits do not exist. Show all the relevant algebra in detail and use limit theorems covered in this course. Correct answer with incorrect reasoning (or algebra) gets no credit!

10 (a) $\lim \left(\dfrac{5^{n+3} + 8^{n+1}}{5^n + 8^n} \right)$

10 (b) For $0 < a < 1$, $\lim(n \cdot a^n)$

10 (c) For $0 < b < 1$, $\lim \left(b^n \cdot (-1)^n \cdot \sin\left(\dfrac{n\pi}{2}\right)\right)$

10 (d) $\lim \left(\dfrac{1}{2^{n \cdot \sin(n\pi/2)}} \right)$

March 18^{th}, 2014

MTH 408-01 TEST 2 **F. Hindeleh**

This Test has 3 questions, for a total of 60 points.
Show all your work in details.

Name(s) : ______________________________

1. For each of the following TRUE/FALSE questions give a complete and precise justification for your answer. Correct answers with no justification /or incorrect justification get no partial credits (so avoid guessing or giving justification that does not make sense to you).

10 (a) Let $(x_n), (y_n)$ be two sequence. Then $\lim(x_n + y_n) = \lim x_n + \lim y_n$

10 (b) A sequence x_n is not bounded if and only if there exists an $M \in \mathbb{R}^+$ such that for every $n \in N$, $|x_n| \geq M$.

10 (c) If (x_n) is a bounded sequence, then (x_n) has a convergent subsequence (x_{n_k}).

2. Let (x_n) be a sequnce of the reals. Complete each of the following definition.

10 (a) (x_n) converges to L if and only if

10 (b) $\lim(x_n) = -\infty$ if and only if

10 3. Explain why the following bounded sequence diverges. For all $n \in N$,

$$x_n = \cos\left(\frac{n\pi}{3}\right)$$

April 11^{th}, 2014

MTH 408-01 FINAL TAKE HOME PORTION **F. Hindeleh**

This Test has 3 questions, for a total of 55 points.
Show all your work in details.
The take home portion is due Wed 4/23/2014 at 8:00 AM

Name(s) : ____________________

[15] 1. We know that $f(x) = \cos(1/x)$ is continuous at every point in the interval $(0, 1)$. Show that $f(x)$ is **not** uniformly continuous on the same interval.

[20] 2. The sequence (x_n) is defined as follows: $x_1 = -4, x_2 = 1$, and for all $n \geq 3, x_n = \dfrac{1}{5}x_{n-1} + \dfrac{4}{5}x_{n-2}$. Show that (x_n) converge and find its limit.

[20] 3. Determine the following limit as use $\varepsilon - \delta$ definition to prove it.

$$\lim_{x \to 0} \frac{\sqrt{1+2x} - \sqrt{1+3x}}{x + 2x^2}$$

This Test has 4 questions, for a total of 90 points.
Show all your work in details.

Name(s) : ______________________________

1. For each of the following TRUE/FALSE questions give a complete and precise justification for your answer. Correct answers with no justification/or incorrect justification get no partial credits (so avoid guessing or giving justification that does not make sense to you).

10 (a) Let (x_n) be sequence of real numbers, such that $(|x_n|)$ converges. Then (x_n) converges.

10 (b) Let $f : A \to \mathbb{R}$. We say that $\lim_{x \to c} f(x) = L$ if and only if there exists a sequence (x_n) such that for all $n \in \mathbb{N}, x_n \in A$, and $\lim x_n = c$ and $\lim f(x_n) = L$.

10 (c) Let $a, b \in \mathbb{R}$. If $f : (a, b) \to \mathbb{R}$ is continuos then f is bounded on (a, b).

10 (d) If $x > 0$ then there exists $n \in \mathbb{N}$ such that $0 < \frac{1}{n} < x$.

2. Let (x_n) be a sequence of the reals, $A \subseteq \mathbb{R}$, $f : A \to \mathbb{R}$, and c is a cluster point of A . Complete each of the following definition.

10 (a) (x_n) is unbounded if and only if

10 (b) $\lim_{x \to c} f(x) = L$ if and only if

10 (c) f is uniformly continuous on A if and only if

10 3. Explain why $\lim_{x \to 2}(5x + 1) \neq 10$. [Hint: Use the negation of the definition and DO NOT show$\lim_{x \to 2}(5x + 1) = 11$].

10 4. Solve the following inequality. $|2x+3| \geq |x|+1$

Mathematics 409
Advanced Calculus II

Section 01
Winter 2014

TR 11:30 - 12:45 A.M. MAK A-2-167.

Instructor: Dr. Firas Hindeleh
MAK A-2-116
(616) 331-3739
`hindelef@gvsu.edu`
Office Hours: TWR 2:00 - 2:50 P.M.,
or by appointment

Prereqisites: Math 227 and MTH 408.

Text: *Math 409 Notes and Exercises Course-Pack*, Winter 2014 edition, by A. Tefera.

Course Content: Key topics to be studied include *differentiation, Riemann and Lebesgue integrals, and sequence and series of functions.*

Course Goals: This course is a continuation of MTH 408 and it introduces students to the fundamental concepts and techniques of real analysis. In every topic we study, we develop the ability to think deductively, analyze mathematical situations, and extend ideas to a new context. An ability to read and write proofs will be stressed. Upon completion of this course, students will be able to:

- Define and determine differentiability of a function.
- Apply the Mean Value Theorem, Rolles Theorem and related results to solve a variety of problems.
- Define and determine the Riemann integrablity of a function on a given interval.
- Comprehend the concepts and properties of measureable sets and Lebesgue Measure.
- Define and determine the Lebesgue integrablity of a function.
- Explain the difference between Riemann and Lebesgue Integrations.
- Comprehend the various definitions and properties related to the convergence of sequence and series of functions and use them to prove and derive related results.
- Explain for what type of functions and under what conditions that:
  - $\lim \sum_n f_n(x)\,dx = \sum_n \lim f_n(x)\,dx$,
  - $\lim \int_{[a,b]} f_n(x)\,dx = \int_{[a,b]} \lim f_n(x)\,dx$,

- $\displaystyle\int_{[a,b]} \sum_n f_n(x)\,dx = \sum_n \int_{[a,b]} f_n(x)\,dx$,
- $\displaystyle\lim \sum_n f_n'(x)\,dx = \sum_n \lim f_n'(x)\,dx, \ldots$

Grading: Your final grade will be determined in the following way:

Homework	35%
In class activities	20%
Exams	30%
Final Exam	15%

The final grade assigned to you will not be lower than that computed in this way. However, I reserve the right to raise your grade if you are an active, thoughtful participant in class and there is evidence you are working consistently and with care.

Homework: You will receive homework assignments every week through Bb. Late homework is accepted with a 20% off penalty. You are encouraged to work on your homework exercises in groups of two in which case one write up per group is sufficient. The importance of working through all the problems carefully cannot be overstated: working problems is the absolute best way to learn mathematics. Homework should be written according to proof writing guidelines you learned in MTH 210. See guidelines for proof writing document.

Exams: There will be two midterms scheduled for Tuesdays Feb 11, and Mar 25. There might be a take home portion that will be due on the same exam day.

Note well: No makeup exams will be given. If you are unable to attend an exam, it is your responsibility to notify me *prior* to the exam so that suitable arrangements may be made.

Final Exam: The final exam will be Wednesday, April 23 from 12:00 - 1:50 P.M in MAK A-2-167.

Final grade: Shown below is the percentage required to obtain a particular grade:

Grade	D	D-	C-	C	C+	B-	B	B+	A-	A
Percentage	60.0	67.0	70.0	73.0	77.0	80.0	83.0	87.0	90.0	93.0

Drop Date: The deadline for withdrawing from this course is Friday, March 7 at 5 P.M.

Expectations:

Attendance: Attendance is expected and critical to your success. Please be on time to class. You are responsible for all announcements made in class concerning material covered, assignments or anything else relevant to the course.

Academic Honesty: Some of the work in this course will be done collaboratively. It is also recommended that you form a study group to assist in learning the material and completing daily homework problems. However, it is important for you to understand that there is a difference between collaboration and plagiarism. Collaboration requires you to contribute to solutions and to think when you write. Representing someone else's work, no matter how small, as your own is plagiarism, a serious offense that will be met with a grade of zero and possible action under

GVSU's guidelines. You are expected to show integrity in all your work and to encourage the same in your colleagues.

Mathematical Communication: One important part of mathematics is its emphasis on the clear and careful presentation of reasoning. This includes clearly stating the problem, making important observations in complete sentences, writing additional thoughts to clarify symbolic expressions, and showing a clear overall progression from problem to solution. The quality of your presentation and writing will count in all of your graded work.

Work load: To be successful in this course, you will need to work hard and consistently. A good rule of thumb is that you should spend at least two hours working outside of class for every hour of class time. Besides working on homework and other assignments, you should also keep a well-organized record of all your study notes and completed problems for future reference. In spite of the fact that we will discuss the most important concepts in detail during class, you will be expected to learn and assimilate many other ideas independently. Please take advantage of my office hours to discuss with me any problems you are having.

Participation: In every class meeting, you will be expected to participate actively and to share your understanding with the class. To do so effectively, you must come to class prepared. I expect that everyone will share in this important aspect of our learning process.

Feedback: I am always happy to discuss with you any thoughts you have about the course including both your performance and mine. Please let me know if you think something could be better or if you like something that we are doing.

Student Concerns: Any student who requires accommodation because of a physical or learning disability must contact Disability Support Resources (`http://www.gvsu.edu/dsr`) at (616) 331-2490 as soon as possible. After you have documented your disability, please make an appointment or see me during office hours to discuss your specific needs.

Any student needing academic accommodations beyond those given to the entire class please be advised that the University's Office of Disability Support Services (DSR, ext. 12490) is available to all GVSU students. It is the student's responsibility to request assistance from DSS.

MTH 409-01 | **HOMEWORK 2** | Jan 16^{th}, 2014 **F. Hindeleh**

This Homework has 4 questions, for a total of 70 points.
Turn one writeup per group on Thursday January 23. **Proofs must follow writing guidelines from MTH 210**.

Name(s) : ______________________________

[10] 1. Let f be a differentiable function at x_0 and g is differentiable at $f(x_0)$. Show that $g \circ f$ is differentiable at x_0 and that $\big(g \circ f\big)'(x_0) = g'\big(f(x_0)\big) \cdot f'(x_0)$.

[10] 2. Using the Mean Value Theorem prove that for all $a, b \in \mathbb{R}$ such that $0 < a < b$,

$$\left(1 - \frac{a}{b}\right) < \ln\left(\frac{b}{a}\right) < \left(\frac{b}{a} - 1\right).$$

[10] 3. (a) Let f be differentiable on (a, b). Prove that if for all $x \in (a, b), f'(x) \neq 0$, then f has at most one zero in (a, b).

[10] (b) Recall that a number c is said to be a *fixed point* of a function f if $f(c) = c$. Show that if f is differentiable on (a, b) and for all $x \in (a, b), f'(x) < 1$, then f has at most one fixed point in (a, b).

4. For each of the following TRUE/FALSE questions, give complete and precise justification for your answer. Correct answers with no justification get no partial credits.

[10] (a) There exists a differentiable function f that satisfies $f(0) = 2, f(2) = 5$, and for all $x \in (0, 2), f'(x) \leq 1$.

[10] (b) There exists a differentiable function f that has the value 1 one when $x = 1, 2$, and 3 and $f'(x) = 0$ only when $x = -1, 3/4$ and $3/2$.

[10] (c) Given any differentiable function f on $\mathbb{R}$ that satisfies $f(0) = 1, f(1) = 2, f(3) = 1$, necessarily $'$ takes the values $-1/2, 0$, and $5/12$.

This Homework has 3 questions, for a total of 100 points.
Turn one writeup per group on Thursday March 27. **Proofs must follow writing guidelines from MTH 210**.

Name(s) : ____________________

1. For each sequence of functions $\big(f_n(x)\big)$, find the pointwise limit function $f(x)$ then determine if the convergence is uniform or not on A

15 (a) $f_n(x) = \frac{1}{nx+1}, A = [0, 1]$.

15 (b) $f_n(x) = \frac{x^{2n}}{1+x^{2n}}, A = [-1, 1]$.

2. Let $f_n(x) = \frac{nx}{1+nx^2}$.

10 (a) For all $x \in (0, \infty)$, find the pointwise limit of (f_n).

10 (b) Is the convergence uniform on $(0, \infty)$? Justify your answer.

10 (c) Is the convergence uniform on $(0, 1)$? Justify your answer.

10 (d) Is the convergence uniform on $(1, \infty)$? Justify your answer.

30 3. Let $g_n(x) = \dfrac{nx + \sin(nx)}{2n}$. Find the point wise limit of (g_n) on $\mathbb{R}$. For $a > 0$, is the convergence uniform on $[-a, a]$? Is the convergence uniform on $\mathbb{R}$? Justify your answers.

This Homework has 3 questions, for a total of 50 points.
Turn one writeup per group on Tuesday April 15. **Proofs must follow writing guidelines from MTH 210**.

Name(s) : ______________________________

10 1. (a) Show that $\lim_{n\to\infty} \int_1^2 e^{-nx^2}\, dx = 0$.

10 (b) For all $0 < a < 1$, evaluate $\lim_{n\to\infty} \int_a^{\pi} \frac{\sin(nx)}{nx}\, dx$.

10 2. Show that $f(x) = \sum_{n=0}^{\infty} 4^n \ln\left(1 + \frac{1}{5^n x}\right)$ is continuous on $[a, \infty)$ where $a > 0$.

[**Hint:** Show that for all $y \geq 0, \ln(1 + y) \leq y$].

3. Find the radius and interval of convergence of each of the following power series.

10 (a) $f(x) = \sum_{n=0}^{\infty} n!x^n$.

10 (b) $f(x) = \sum_{n=0}^{\infty} \frac{n!}{n^n} x^n$.

MTH 409-01 | **Test 2 Take Home** | March 18^{th}, 2014 | **F. Hindeleh**

This test has 2 questions, for a total of 55 points.
Answer the questions in the spaces provided on the question sheets. **Show all your work in details**.

Name : ____________________

1. Let $f(x) = \begin{cases} 1 & \text{if } x = 1 \\ x+1 & \text{if } 1 < x < 4 \\ 6 & \text{if } x = 4 \end{cases}$.

10 (a) Explain why $f \in \mathcal{R}[1,4]$.

15 (b) Use Theorem R.5.2 to compute $\int_1^4 f(x)\,dx$.

2. Let $f_n(x) = \dfrac{x^n}{1+x^n}$ be sequence of functions.

10 (a) Show the pointwise limit function

$$f(x) = \begin{cases} 0 & \text{if } x < 1 \\ 1/2 & \text{if } x = 1 \\ 1 & \text{if } x > 1 \end{cases}.$$

20 (b) Show that the convergence in part (a) is not uniform on $[0, b]$ where $0 < b < 1$ and not uniform on the interval $[c, \infty)$ where $c > 1$.

March 25^{th}, 2014

MTH 409-01 TEST 2 **F. Hindeleh**

This test has 1 questions, for a total of 45 points.
Answer the questions in the spaces provided on the question sheets. **Show all your work in details**.

Name : ______________________________

1. **Theorem S.1.6** Let (f_n) be sequence of functions defined on $I := [a, b]$. Suppose that

   (i) For $n \in \mathbb{N}, f_n$ is differentiable on I.

   (ii) There exists an $x_0 \in I$ such that the sequence $\Big(f_n(x_0)\Big)$ converges, and

   (iii) (f_n') converges uniformly to g on I.

   Then

   (a) (f_n) converges uniformly on I to a function f, and

   (b) f is differentiable on I and for all $x \in I, f'(x) = g(x)$.

   We proved part (a) in class. To prove part (b) you may assume the following results are true

   - $$\left|\frac{f(x) - f(x_0)}{x - x_0} - g(x_0)\right| \leq \left|\frac{f(x) - f(x_0)}{x - x_0} - \frac{f_n(x) - f_n(x_0)}{x - x_0}\right| + \left|\frac{f_n(x) - f_n(x_0)}{x - x_0} - f_n'(x_0)\right| + |f_n'(x_0) - g(x_0)|.$$

   - For $x > x_0$ be arbitrary in I, there exists a $c \in (x_0, x)$ such that

   $$f_m'(c) - f_m'(c) = \frac{[f_m(x) - f_n(x)] - [f_m(x_0) - f_n(x_0)]}{x - x_0}$$

   - There exists an $N_1 \in \mathbb{N}$ such that for all $m, n \geq N_1, |f_m'(c) - f_m'(c)| < \frac{\epsilon}{3}$. Hence for all $m, n \geq N_1$ and for all $x \in I$

   $$\left|\frac{f_m(x) - f_m(x_0)}{x - x_0} - \frac{f_n(x) - f_n(x_0)}{x - x_0}\right| < \frac{\epsilon}{3}.$$

15 (a) Show that for all $n \geq N_1$

$$\left|\frac{f(x)-f(x_0)}{x-x_0}-\frac{f_n(x)-f_n(x_0)}{x-x_0}\right|<\frac{\epsilon}{3}.$$

10 (b) Show that there exists an $N_2 \in \mathbb{N}$ such that for all $m \geq N_2$, $|f'_m(x_0)-g(x_0)|<\frac{\epsilon}{3}$.

10 (c) Let $N=\max\{N_1,N_2\}$. Find a $\delta>0$ such that whenever $0<|x-x_0|<\delta$ and $x \in I$, we have

$$\left|\frac{f_N(x)-f_N(x_0)}{x-x_0}-f'_N(x_0)\right|<\frac{\epsilon}{3}$$

10 (d) Complete the proof of part (b) of Theorem S.1.6.

April 23^{rd}, 2014

MTH 409-01 FINAL EXAM **F. Hindeleh**

This test has 11 questions, for a total of 350 points.
Answer any combination of questions or parts that adds up a **maximum of 200 points**. Turn in your exam in class on Wednesday April 23, 2014 at 2:00pm

Name : ______________________________

10 1. Evaluate $\lim_{x\to 0^+} x \cdot \ln(\sin x)$.

10 2. Find $f'(0)$ if $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$.

3. Determine whether $f \in \mathcal{R}[0,1]$ in each case, and justify your answer.

10 (a) $f'(0)$ if $f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$.

10 (b) $f'(0)$ if $f(x) = \begin{cases} \sin\left(\frac{1}{x}\right) & \text{if } x \notin \mathbb{Q} \\ 0 & \text{otherwise} \end{cases}$.

4. For each of the following TRUE/FALSE questions, give complete and precise justification for your answer. Correct answers with no justification get no partial credits.

10 (a) Let $g \in \mathcal{B}[0,1]$. If for all $a \in (0,1), g \in \mathcal{R}[a,1]$, then $g \in \mathcal{R}[0,1]$.

10 (b) Let $g \in \mathcal{B}[0,1]$. If $g \in \mathcal{R}[0,1]$, then for all $a \in (0,1), g \in \mathcal{R}[a,1]$.

10 (c) Assume that $f \in \mathcal{R}[a,b]$ and for all $x \in [a,b], f(x) \geq 0$. If for an infinite number of points $x \in [a,b], f(x) > 0$, then $\int_a^b f(x)\,dx > 0$.

10 (d) If $\int_a^b f(x)\,dx > 0$, then there exists an interval $[c,d] \subseteq [a,b]$ and a $\delta > 0$ such that for all $x \in [c,d], f(x) \geq \delta$.

10 (e) If f is a non-negative function and continuous on $[a,b]$, and there exists an $x_0 \in [a,b]$ such that $f(x_0) > 0$, then $\int_a^b f(x)\,dx > 0$.

10 (f) There exists a non-negative function f defined on a closed and bounded interval $[a,b]$ with the property that $f \in \mathcal{R}[a,b]$ and there exists an $x_0 \in [a,b]$ such that $f(x_0) > 0$, but $\int_a^b f(x)\,dx = 0$.

5. Let $f(x) = \begin{cases} 4 & \text{if } x \in [0,2] \cap \mathbb{Q} \\ 3 - x & \text{if } x \in [0,2] \cap \mathbb{Q}^c \end{cases}$.

10 (a) For all partitions $\mathcal{P}$ of $[0,2]$, find $U(f,\mathcal{P})$.

10 (b) Determine the value of $\overline{\int_0^2} f$.

10 (c) For a regular partition $\mathcal{P}_n$ of $[0,2]$ with n subintervals, find $L(f,\mathcal{P}_n)$.

10 (d) Determine the value of $\underline{\int_0^2} f$. Prove your result.

15 6. Prove that if f and g are continuous on $[a, b]$ and for all $x \in [a, b], g(x) \neq 0$, then there exists a $c \in (a, b)$ such that $\int_a^b f(x)g(x)\,dx = f(c)\int_a^b g(x)\,dx$.

Hint: Let $G(x) := \int_a^x f(t)g(t)\,dt$ and $F(x) := \int_a^x g(t)\,dt$. Apply the generalized (Cauchy's) MVT.

15 7. (a) If f and g are continuous on $[a, b]$ and if f is non-negative on $[a, b]$ and g is decreasing and positive on $[a, b]$, then there exists a $c \in (a, b)$ such that

$$\int_a^b f(x)g(x)\,dx = g(a)\int_a^c f(x)\,dx.$$

Hint: For $x \in [a, b]$ define $F(x) = g(a)\int_a^x f(t)\,dt$. Show that for all $t \in [a, b]$, $0 \leq f(t)g(t)$. Apply IVT on $F(x)$.

15 (b) Prove that there exist values $a, b \in [0, 1]$ such that

$$\int_0^1 \frac{\sin(\pi x)}{x^2+1}\,dx = \frac{2}{\pi(a^2+1)} = \frac{\pi}{4}\sin(\pi b).$$

10 8. Let $f(x) = \sum_{n=1}^{\infty} \frac{\sin(nx)}{n^3}$. Show that $\int_0^\pi f(x)\,dx = 2\sum_{n=1}^{\infty} \frac{1}{(2n-1)^4}$.

9. Find suitable coefficients (a_n) so that the resulting power series $\sum a_n x^n$

10 (a) converges absolutely for all $x \in [-1, 1]$ and diverges off this interval.

10 (b) converges conditionally at $x = -1$ and diverges at $x = 1$.

10 (c) converges conditionally at both $x = 1, -1$.

10 (d) Is it possible to find an example of a power series that converges conditionally at $x = -1$ and converges absolutely at $x = 1$?

15 10. (a) Let $A \subseteq \mathbb{R}$. Prove that $m^*(A) = 0$ if and only if for every $B \subseteq \mathbb{R}$, $m^*(A \cup B) = m^*(B)$.

15 (b) Prove that for any $A \subseteq \mathbb{R}$ and any $\epsilon > 0$, there exists a sequence of open intervals I_n with the property that $A \subseteq \bigcup_{n=1}^{\infty} I_n$ and $m^*\left(\bigcup_{n=1}^{\infty} I_n\right) \leq m^*(A) + \epsilon$.

15 (c) Prove or disprove that any subset of the irrational numbers is measurable.

15 (d) For all $n \in \mathbb{N}$ let $E_n = \{x\mathbb{R} : \frac{1}{2^n} \leq x < \frac{1}{2^{n-1}}\}$. Let $E = \bigcup_{n=1}^{\infty} E_n$. Determine whether E is measurable. If it is measurable, find the exact value of its measure.

11. Recall from set theory that the difference of two sets E, F is defined by

$$E - F = E \cap F^c,$$

and the symmetric difference of E and F denoted by $E \Delta F$ is defined by

$$E \Delta F = (E - F) \cup (F - E).$$

10 (a) Prove that if E and F are measurable then $E - F$ and $E \Delta F$ are also measurable.

15 (b) Let E and F be measurable sets with finite measure. Prove that $m(E \Delta F) = 0$ if and only if $m(E) = m(E \cap F) = m(F)$.

15 (c) If E is measurable and $m^*(E \Delta F) = 0$, then show that F is measurable.

15 (d) If E_n is a sequence of sets with the property that for all $n \in \mathbb{N}, m^*(E_n) = 0$, then show that $\bigcup_{n=1}^{\infty} E_n$ is a measurable set and has a measure zero.

Thank you for taking MTH 409

GOOD LUCK!