Investigating student understanding in intermediate mechanics: Identifying the need for a tutorial approach to instruction

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The conceptual understanding and reasoning skills of advanced undergraduates as they make the transition from a traditional sequence in introductory calculus-based physics to their first course in upper-level mechanics are probed. The results thus far are consistent with findings from other investigations in upper-division courses, which indicate that persistent difficulties with fundamental concepts can hinder meaningful learning of advanced topics. To address this problem, the tutorial approach developed at the University of Washington has been adapted and incorporated into the intermediate mechanics course at Grand Valley State University. This modification has produced promising results.
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I. INTRODUCTION

Ongoing investigations in physics education research have been conducted over the past several years at Grand Valley State University in introductory-level courses and in selected upper-level undergraduate courses. The scope of the research has been expanded to include the junior-level course in intermediate mechanics. This particular study is motivated in part by prior investigations of student understanding of other advanced topics, including quantum physics, relativity, and thermodynamics.[1–3] Emerging from such investigations is a growing body of evidence suggesting that difficulties with fundamental concepts are not addressed after standard lecture instruction at or beyond the introductory level.

A primary objective of this study has been to probe the conceptual understanding and reasoning skills of advanced undergraduates as they make the transition from a traditional sequence in introductory calculus-based physics to their first course in upper-level mechanics. Although recent investigations have begun to explore the learning of related topics in the engineering sciences,[4] few studies have been reported in the context of upper-level mechanics courses for physics majors.[5,6] The results presented here are intended to help address this gap in the current research base, as well as to motivate the need for a modified approach in teaching intermediate mechanics.

II. CONTEXT OF INVESTIGATION

The research reported in this paper proceeds from the fundamental assumption that an important goal of physics instruction is to develop the ability to analyze, model, and predict the outcome of physical phenomena. Thus an essential objective of our research is to elicit and observe the reasoning, intuition, and resources that students bring to bear as they make predictions about specific situations. In this study the primary method of research is the analysis of student responses to carefully designed written questions. These questions are usually qualitative in nature, for example, asking for comparisons between (greater than, less than, equal to) or rankings among various quantities. In their responses the students are required to explain the reasoning they use to determine their answers.[7] The results from these questions are supplemented by informal observations of students in the classroom and the analysis of other written work submitted as part of the regular course requirements.

The target group in this investigation consists of students enrolled in the junior-level intermediate mechanics course at Grand Valley. The course meets for four 50-min lecture periods each week. The participants in this study were 26 students, almost exclusively physics majors and minors, who took intermediate mechanics during the fall semesters from 2001 to 2003. The author of this paper served as the instructor.

The emphasis of the junior-level course is the extension and application of concepts from introductory mechanics. After an in-depth review of topics from introductory mechanics (kinematics, Newton's laws, conservation of energy), students investigate a variety of physical situations for which Newton's second law takes the form of a differential equation that is then solved by the instructor, the textbook, or the students themselves. Course topics include velocity-dependent forces (for example, air resistance), linear and nonlinear oscillations, separable forces, conservative forces, orbital dynamics, and noninertial reference frames. The intermediate mechanics course does not cover variational methods or the Lagrangian or Hamiltonian formulations of classical mechanics; these topics are reserved for the senior-level course in advanced mechanics.

Students also are introduced to new mathematical representations and tools, including phase space diagrams and elements of vector calculus (for example, the del operator, gradient, and curl). Many students take intermediate mechanics concurrently with a junior-level mathematics course in vector calculus and applied analysis, although this course is neither a prerequisite nor a co-requisite. Students receive additional instruction in vector calculus in the junior-level electricity and magnetism course, which they usually take after intermediate mechanics.

III. STUDENT UNDERSTANDING OF TOPICS IN INTERMEDIATE MECHANICS

Presented here are the results from written problems identical or similar to others that have been fruitful in eliciting student reasoning patterns at the introductory level. The problems were included on ungraded quizzes or pretests given in lecture. Unless otherwise noted, the pretests were administered after standard instruction on the relevant topics, whether at the advanced or introductory level.

For the purposes of this paper, most of the discussion will be on the data obtained on pretests that probe student understanding of Newton's laws, velocity-dependent forces, and conservative forces. The results, in conjunction with informal observations of students during instruction, were used to identify conceptual and reasoning difficulties that hindered meaningful learning of numerous topics in mechanics.

A. Example #1: Newton's second law and velocity-dependent forces

Throughout the intermediate mechanics course students are expected to apply Newton's second law to set up and solve differential equations for various physical systems. To this end, an important skill that the students are assumed to have acquired from their introductory mechanics course is the ability to draw and interpret free-body diagrams, that is, to isolate an appropriate object (or set of objects) as a system and identify all of the forces exerted on the system. Furthermore, physics majors are expected to be adept not only at using free-body diagrams, but also at recognizing when they are helpful in solving problems.

1. Description of pretest tasks

A pretest was designed to explore the ability of students to apply Newton's second law in situations involving air resistance, a context usually not covered in the introductory course at Grand Valley. The pretest was administered after relevant lecture instruction on air resistance as a velocity-dependent force. The pretest consisted of two problems, the *Skydiver problem* and the *SuperBall problem*. The students were told explicitly not to neglect air resistance in completing the pretest.

The Skydiver problem asked students to draw free-body diagrams for a skydiver at two instants: (i) shortly after jumping from a plane, and (ii) shortly after the diver begins to descend with constant (terminal) speed. The students are also asked to compare the net force on the skydiver at instant (i) to that at instant (ii) and to explain their reasoning. The students are expected to recognize that at instant (ii) both the acceleration and the net force are equal to zero, making the net force at instant (i) larger than that at instant (ii). In addition, the free-body diagram for the skydiver at instant (ii) must include a force of air resistance that is equal and opposite to the gravitational force.

The SuperBall problem (see Fig. 1) is a task similar to the Skydiver problem except students must consider both upward and downward motion in the presence of air resistance. The students are told that a SuperBall is dropped vertically downward and that the speed of the SuperBall just before reaching the floor is the same as its speed just after leaving contact with the floor. The students are asked to determine whether the acceleration of the SuperBall is larger (1) just before it reaches the floor, (2) just after it bounces off the floor, or (3) if it is the same magnitude at both instants. The students are not given explicit instructions to draw free-body diagrams. Students are expected to recognize that the force exerted by the surrounding air has the same magnitude at both instants. However, the net force is larger just after it bounces off the floor, when both the weight and air resistance are directed down. By Newton's second law, the acceleration is larger just after the SuperBall bounces off the floor.

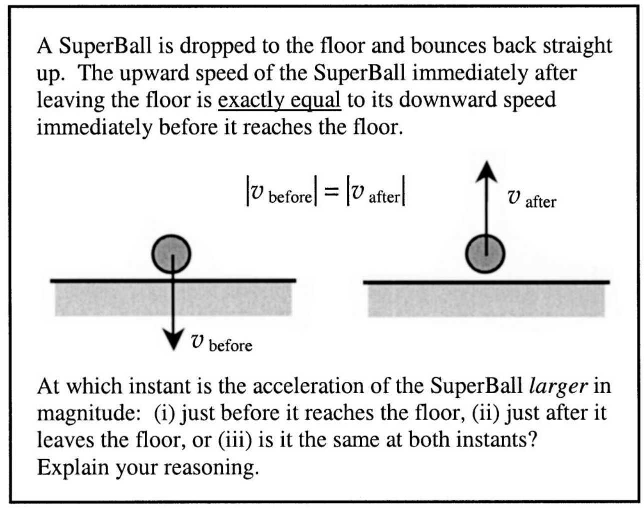

Fig. 1. The *SuperBall problem*. This problem was used on a pretest given in intermediate mechanics after lecture instruction on air resistance.

2. Pretest results

Essentially all of the students drew correct free-body diagrams for the Skydiver problem. Nearly all correctly stated that the net force on the skydiver is larger in magnitude just after commencing the jump. More than 80% (21 of 26) gave complete and correct explanations. In contrast, only about one-third of the same students (10 of 26) gave a correct answer to the SuperBall problem with correct reasoning. The most common incorrect response, given by over 40% of the students (11 of 26), was to state that the acceleration of the ball had the same magnitude just before and just after hitting the floor.

Despite the strong performance on the Skydiver problem, students employed a variety of incorrect kinematical and dynamical arguments to support their answers. For example, some students did not attempt to solve the SuperBall problem by using Newton's second law. Instead, these students appeared to base their answer merely on the fact that the ball had the same speed at both instants. Some gave explanations similar to that described in the following student response:

> "Acceleration is derived from velocity, which is equal in magnitude in both cases, so acceleration must be the same at both instants."

Responses such as this suggest the same sort of confusion between velocity and acceleration that has been documented among introductory physics students after traditional instruction in kinematics.[8] Additional evidence of this difficulty among the intermediate mechanics students has come from other pretests in the context of one-dimensional motion. For example, when asked to consider motion that involved a turnaround point (for example, an object undergoing simple harmonic motion or a ball rolling up and down an inclined plane),[9] many students incorrectly stated that the acceleration would be equal to zero at the turnaround.

Many students apparently recognized the relevance of Newton's second law in answering the SuperBall problem. Some concluded correctly that the force of air resistance had the same magnitude just before and just after the collision with the floor. However, many failed to recognize, or neglected to take into account, how the direction of this force affected the net force on the ball. For example, using

"f_{BA}(i)" and "f_{BA}(ii)" to represent the frictional force by the air at the two designated instants, one student explained: "If we assume the ball has the same velocity just before and just after, [then] $f_{BA}(\mathrm{i})=f_{BA}(\mathrm{ii})$, so the two balls have the same F_{net} and thus the same a."

It should be noted that the Skydiver problem, which most students answered correctly, explicitly asked for the relevant free-body diagrams while the SuperBall problem did not. On the latter problem six students drew free-body diagrams on their own, and all six gave complete and correct responses. In contrast, only one-fifth of the remaining students answered correctly.

3. Discussion of related difficulties

The qualitative nature of the SuperBall problem helped reveal the presence of difficulties with acceleration and Newton's second law. The intermediate students were generally successful in recognizing that acceleration must point in the same direction as the net force. However, as seen by the examples discussed above, many demonstrated serious conceptual and reasoning difficulties in applying an operational definition of acceleration, in describing the net force (whether verbally or mathematically), and in recognizing the cause–effect relationship between net force and acceleration. Additional evidence for such difficulties was observed later in the course on other written problems involving velocity-dependent forces.

For example, on homework and exam problems, students frequently had difficulty translating information from a correct free-body diagram to a differential equation of motion. For problems in which air resistance could be expressed as a linear function with respect to speed (for example, for spherical water or oil droplets), many students wrote down correct equations for the vertical motion: $m(dv/dt)=-mg-\alpha v$ (where up is taken to be the positive direction). The second minus sign indicates that air resistance always is opposite in direction from the (signed) velocity. However, for situations that call for the quadratic formulation instead (for example, for larger objects such as marbles or softballs), many students failed to recognize that the overall sign depends upon the direction of the velocity. That is, although a term of the form $-\beta v|v|$ is generally correct, many believed instead that a term of the form $-\beta v^2$ would be valid even for objects moving downward (if $v<0$).

Other reasoning errors related to those elicited by the SuperBall problem arose in the context of damped harmonic motion. For undamped oscillators, students readily accepted the fact that the maximum speed of the oscillator occurs when it passes equilibrium ($x=0$). However, many students inappropriately generalized this result to the underdamped case. They required guidance to recognize that, during each cycle of the motion, the oscillator experiences zero net force (and thus attains a maximum speed) before reaching $x=0$. Understanding this subtlety is important for students to construct and interpret phase space diagrams for underdamped oscillators, whose trajectories cross the x axis at right angles but cross the $\dot{x}$ axis obliquely.

B. Example #2: Conservative forces and potential energy

Midway through the intermediate mechanics course students gain experience with vector calculus by analyzing vector force fields and potential energy functions in three dimensions. Students are led to build upon their previous experience with gravitational and electric fields by being shown that any force field $\mathbf{F}(\mathbf{r})$ is conservative if and only if it has zero curl ($\nabla\times\mathbf{F}=0$) at all locations. It is then proved that any conservative force $\mathbf{F}(\mathbf{r})$ has a corresponding potential energy function $U(\mathbf{r})$ such that $\mathbf{F}=-\nabla U$.

Assessing the extent to which students grasped these ideas from their previous math and physics courses might make it possible to predict where future difficulties might arise. For instance, would students who have covered Maxwell's equations in calculus-based electricity and magnetism courses be able to extend their knowledge of the electric field and the electric potential from electrostatics (where $\nabla\times\mathbf{E}=0$ and thus $\mathbf{E}=-\nabla V$) to the classical mechanics context? Such an expectation is likely to be unrealistic for at least two reasons. At Grand Valley and other universities, the integral forms of Maxwell's equations are usually treated at the introductory level with much greater depth than are the differential forms. Also, many students have limited (if any) intuition about the curl of a vector field when the topic is introduced in intermediate mechanics. As mentioned in Sec. II, the intermediate mechanics course at Grand Valley is often, but not always, taken concurrently with a junior-level course in applied analysis and vector calculus. As a result, those students who have not yet taken the analysis course receive their first introduction to vector calculus in the mechanics course.

In order to take into account the varying levels of mathematical background, two separate tasks were designed for a pretest on conservative forces and potential energy. The first task posed questions in the context of electrostatics in such a way as to probe qualitative understanding of the relation $\mathbf{F}=-\nabla U$ between potential energy and force. The second task required students to interpret the meaning of a vector curl. Some students would have an insufficient background to attempt the second task, but all could draw on their prior knowledge of electricity and magnetism to answer the first one.

1. Description of pretest task: Equipotential problem

The first pretest task, referred to here as the *Equipotential problem*, is shown in Fig. 2. The students are presented an equipotential contour map on which three locations (A, B, and C) are labeled. (The map given to the students does not include the superimposed vectors shown in Fig. 2.) Two of the marked locations, B and C, are equidistant from a point charge $+Q_0$ located in the region shown on the map. The students are asked two questions regarding an imaginary test charge $+q_{\mathrm{test}}$ placed at each of the three locations. In part A the students are asked to draw an arrow at each labeled location to indicate the direction in which the test charge would move if it were released from rest at that location. In part B they are asked to rank the labeled locations according to the magnitude of the force exerted on $+q_{\mathrm{test}}$ when placed at those locations. As always, the students are asked to explain their reasoning.

Students are expected to recognize that at each location the electric force exerted on $+q_{\mathrm{test}}$ is perpendicular to the equipotential line at that location in the direction of decreasing potential energy. In addition, the relative magnitude of the electric field (or force) at each location is indicated by the proximity of neighboring equipotential lines in the vicinity of that location. The closer the lines are to each other, the

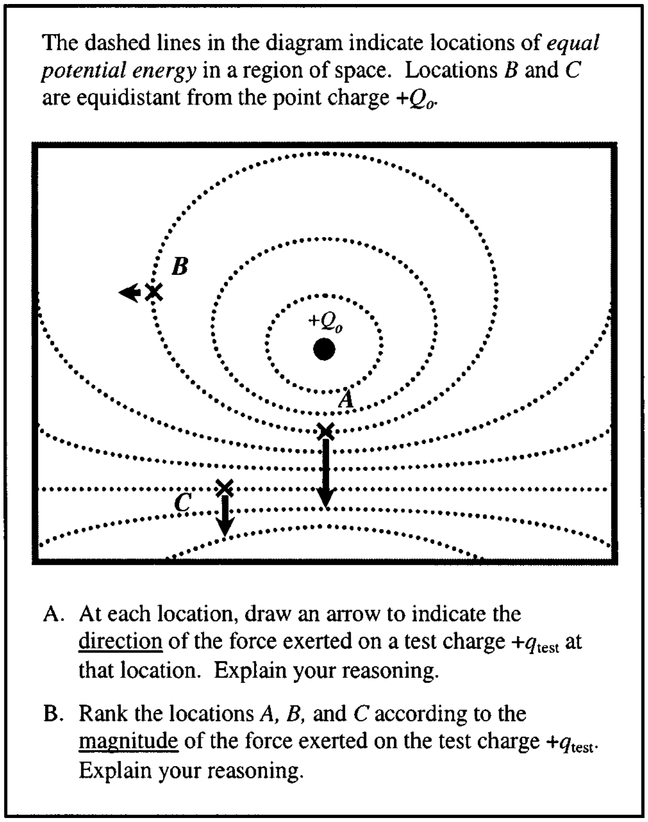

Fig. 2. The *Equipotentials problem* (with qualitatively correct force vectors shown for reference). This problem was used on a pretest given in intermediate mechanics.

stronger the electric field, and hence the stronger the force exerted on the test charge. Figure 2 includes force vectors whose relative magnitudes indicate the intended correct ranking: $F_A > F_C > F_B$.

The equipotentials problem was designed so that a complete and correct response would indicate a working knowledge of the ideas underlying the relationship $\mathbf{F} = -\nabla U$ without requiring familiarity with the gradient as a directional derivative. The problem also was intended to probe student ideas about force and potential energy, not necessarily about electric field or electric potential. Additional difficulties intrinsic to electrostatics were likely to be present.[10] To pinpoint just those difficulties that would arise in a classical mechanics context, several elements of the problem were simplified. For example, the equipotential contours were called "lines of equal potential energy" so as to avoid possible student confusion between electric potential and potential energy. Similarly, the test charge was chosen to be positive so that difficulties discriminating between electric field and electric force would not impede the ability of the students to answer the problem correctly.

2. Pretest results: Equipotential problem

A total of 22 students (not all 26) attempted the equipotentials problem. Very few (4 of 22) answered both parts A and B correctly. In part A, almost all of the students correctly recognized that the force on the positive test charge would generally point away from (rather than toward) the point charge $+Q_0$. However, only 11 of 22 correctly recognized that at each of the labeled locations the force would be perpendicular to the local equipotential lines.

In part B, only 15% of the students gave correct responses with correct reasoning. Some students either neglected to notice that the equipotential contours were not perfectly circular or failed to recognize what that meant physically, that is, that the point charge $+Q_0$ must be accompanied by other charges nearby. These students ranked the magnitudes of the forces at the three locations purely according to the proximity of those locations to the point charge: $F_A > F_B = F_C$. The most common incorrect response in part B (8 of 22) was to give a ranking that would be appropriate for the potential energy at the three locations rather than for the magnitude of the force. As one student responded: "$A = B > C$. A and B lie on the same equal potential line [sic] and C lies at a farther equal potential line."

3. Discussion of results: Equipotential problem

The above results expose another instance of the well-documented difficulty that many students have in distinguishing between a quantity and its rate of change. In this case, the difficulty seems to lie with the potential energy and its spatial rate of change (magnitude of the force). This result is consistent with findings from prior investigations of student understanding of electric potential, electric potential energy, and electric field.[11] Even when students attempt on their own to make an analogy between equipotential diagrams and topographic maps—an analogy emphasized in some textbooks—informal observations in the algebra-based physics course at Grand Valley have revealed a persistent confusion between slope and elevation. These results also strongly resonate with prior research in student understanding of graphs, particularly with difficulties that students often have in discriminating between the slope and height of a graph.[12]

4. Description of pretest task: Vector curl problem

The second pretest task, referred to here as the *Vector curl problem*, was designed to probe student understanding of the curl of a vector force field and its relation to the conservative nature of the force. Students were given four diagrams (see Fig. 3) that graphically represent four different vector fields in the $x-y$ plane.[13] Part A of the problem asked students to identify which vector fields, if any, had zero curl at all locations. On part B the students needed to identify which of the four diagrams could represent a conservative force. On both parts students were asked to explain the reasoning used to determine their answers. Finally, the students were asked to indicate on the pretest whether or not they were familiar with the term "curl."

Asking both parts of the Vector curl problem provided the opportunity to test whether students recognized that any conservative force (such as those illustrated in cases 1 and 3 of the problem) must also have zero curl at all locations. Students might bring to bear any one of several correct lines of reasoning to answer the problem. For example, if a closed path could be found such that the work done over that path is not zero then the force must be nonconservative, and the curl cannot be zero everywhere within the region bounded by the path.[14] Such paths would include any circular path in case 2 that is concentric with the origin and any rectangular path in case 4 whose center is located above or below the x axis.

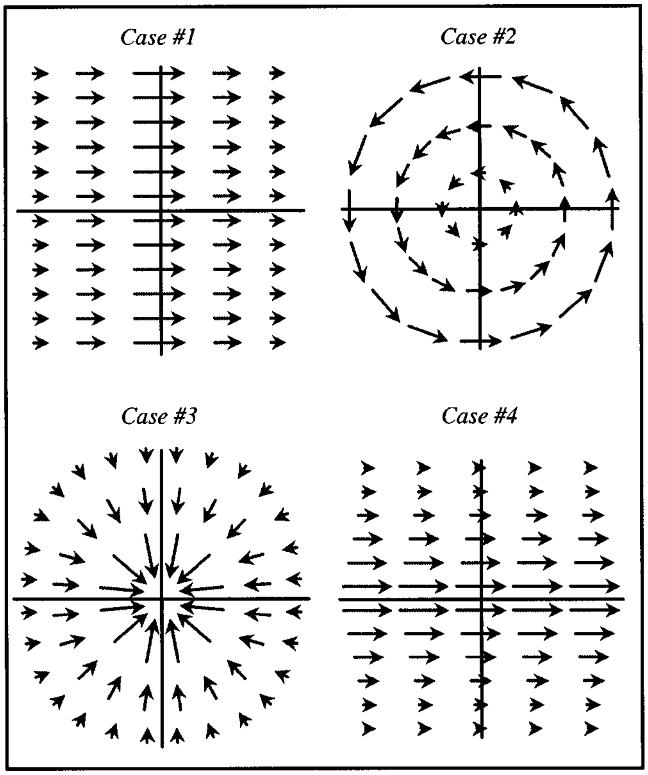

Fig. 3. Illustrations of vector fields mapped in the $x-y$ plane, as shown on the *Vector curl problem*. Part A of the problem asked students to identify which fields had zero curl at all locations. On part B the students needed to identify which of the four diagrams could represent a conservative force. Both parts required students to explain their reasoning.

Alternatively, if it is not possible to draw a set of self-consistent equipotential contours for a particular vector field, then the force must be nonconservative. To be self-consistent, the equipotential lines must not only cross the force vectors at right angles, but each line must correspond to a unique value of the potential energy (to within an additive constant).

5. Pretest results: Vector curl problem

The same 22 students who answered the equipotential problem also attempted the vector curl problem. Almost half of the students (10 of 22) indicated that they had learned about the vector curl prior to taking the pretest. However, only 2 of these 10 students answered all parts of the problem correctly. If we treat the students' answers to each of the four vector fields individually (giving a total of 88 pairs of responses), we find that the overall success rate on parts A and B combined was just over 30% (28 correct out of 88). Ignoring correctness, students gave a consistent pair of responses—stating ''zero curl'' in part A and ''conservative'' in part B, or stating ''nonzero curl'' and ''nonconservative''—only about half of the time (40 of 88). Furthermore, the 10 students who had had prior instruction on vector curls were no more likely to give consistent pairs of answers than were the other students.

On part A, which asked about the curl of each vector field, the percentages of correct responses among the four example vector fields varied considerably. Cases 1 and 2 each yielded a clear majority of correct answers, although the students' explanations were usually incomplete. Most students who gave incorrect answers for cases 3 and 4, including those who had studied the curl prior to taking the pretest, tended to associate the curl of a vector field only with variations in direction of the vectors. For example, many incorrectly stated that the curl would be zero everywhere for case 4, explaining that there is ''no twist to the field'' or the ''force does not change direction.''

Part B of the problem, which asked students to identify which forces were conservative, seemed more difficult than part A. If reasoning is ignored, 55% (12 of 22) or fewer gave a correct answer for any particular case. For case 3, which could be used to illustrate the familiar case of an inverse-square force law, only half of the students correctly stated that that force would be conservative. The lowest success rates occurred for cases 1 and 2. In these cases students often gave explanations suggesting that they associated the conservative nature of a force with variations in magnitude of the force vectors. For example, many students incorrectly believed case 1 represented a nonconservative force due to the ''change in magnitude along the field lines.'' Similarly, case 2 was mistaken for a conservative force because there was ''no change in magnitude along the field lines,'' or that the ''forces [are] equal with equal distances from the origin.''

6. Discussion of results: Vector curl problem

The level of difficulty presented to students by the vector curl problem is not unusual given the fact that many had not yet covered the vector curl before taking the pretest. Intuition regarding the variations in the direction of the force vectors (for example, whether or not there is a ''twist to the field'') is not unexpected. However, the tendency for students to focus on variations in the magnitude of the vectors as a criterion of the conservative nature of a force is surprising. This notion is clearly inconsistent with the intended development of the concept of conservative forces at the introductory level. The results from the pretest are not conclusive; a natural next step would be to conduct individual student interviews to probe in greater detail the nature of student thinking. Yet, these results indicate the presence of difficulties in interpreting the calculations of gradients and curls. Such difficulties would likely persist after all lecture instruction in mechanics and vector calculus.

IV. NEED FOR A TUTORIAL APPROACH TO INSTRUCTION

The prevalence of the difficulties identified thus far in the study strongly suggested the need to modify the instructional approach used in intermediate mechanics. The students apparently required guidance to build a coherent conceptual framework and to infuse deeper meaning into the mathematical formalism encountered in the course. The rapid pace of the course and the wide variety of topics demands that any intervention be of relatively short duration and strongly targeted. The tutorials developed by the Physics Education Group at the University of Washington satisfy these criteria and have been demonstrated to be effective in introductory physics courses. Given the author's experience as a former member of the Physics Education Group and as a contributor to *Tutorials in Introductory Physics*,[15] tutorials seemed to be a natural first choice as a supplement to the lectures. The original context of research and development of the tutorials was the introductory calculus-based course. However, a tutorial approach has been successfully adapted for other student populations.[16]

At the heart of the tutorials are carefully structured worksheets that guide students through the reasoning necessary to overcome specific difficulties identified by research. Students work collaboratively through the tutorials in small groups, and the instructor teaches by questioning rather than by telling. In the intermediate mechanics course, tutorials were used for one or two of the four lecture periods each week, with no significant change to the total time spent on each topic or to the breadth of coverage in the course. Homework problems were assigned to give students the opportunity to generalize and extend their results from the tutorials. Questions on pretests and course examinations are used to measure the prevalence of common difficulties before and after tutorial instruction, providing a means for assessing the effectiveness of the tutorials.[17]

A. Implementation of existing tutorials in the intermediate mechanics course

To address persistent and prevalent difficulties with basic concepts from introductory mechanics, some tutorials were taken directly from Ref. 15. In particular, tutorials on kinematics and Newton's laws[18] were used with the goal of addressing difficulties in relating velocity and acceleration, reinforcing skills in drawing free-body diagrams, and understanding Newton's second law. Results from pretests and post-tests indicate that this goal is being achieved. For example, similar research tasks were given to the Grand Valley intermediate mechanics students after tutorial instruction and to physics graduate students at other universities after traditional undergraduate instruction. On these tasks, which were designed to probe understanding of acceleration in two dimensions, the intermediate mechanics students consistently outperformed the graduate students (65% correct versus 15% correct).[19] This result provides strong evidence that the tutorials played a critical role in helping students develop and apply an operational definition of acceleration.[20]

B. Development of new tutorials in intermediate mechanics

Many of the underlying difficulties identified in Sec. III have their roots in introductory mechanics. However, addressing those difficulties in the contexts usually encountered at the introductory level does not guarantee that these same difficulties will not resurface in the intermediate course. Furthermore, additional difficulties are intrinsic to topics covered specifically at the intermediate level. For example, few students understood or could apply the idea that the work done by a conservative force is path-independent, or that any force with zero curl is conservative. Students frequently needed explicit guidance in assimilating the new concepts to which they were being introduced and in making appropriate connections between the physics and the mathematics (for example, phase space diagrams and vector and scalar fields).

The results from the research described in this paper motivated the incorporation of tutorials as a way to address specific conceptual and reasoning difficulties when they arise during intermediate mechanics. To this end, a set of tutorial-style materials—consisting of pretests, tutorial worksheets, homework problems, and post-tests—has been developed and is being tested at Grand Valley in the intermediate mechanics course. The new tutorials utilize the same overall approach in helping students recognize and address the gaps in their understanding. These materials span the various topics that have comprised the scope of the research, including velocity-dependent forces, linear and nonlinear oscillations, phase space diagrams, conservative forces, accelerating reference frames, and Kepler's laws. The new tutorials have yielded promising results thus far in addressing specific difficulties. The materials continue to be refined and tested on the basis of ongoing research with the intended student populations, both at Grand Valley and at a growing number of test sites.

V. CONCLUSION

In this paper we have presented specific ''snapshots'' of student understanding at various stages during the intermediate mechanics course. The analysis of responses to carefully designed qualitative questions has revealed serious conceptual and reasoning difficulties with basic physics. Some of these difficulties, such as the failure to distinguish between a quantity and its (temporal or spatial) rate of change (for example, between velocity and acceleration, potential energy and force), have proved resistant to change at the introductory level. Results from research conducted in upper-level courses indicate that such basic difficulties, if allowed to persist, can and do inhibit meaningful learning of advanced topics.[1–3]

Despite the typically small class sizes in upper-level courses compared with introductory courses, the results from research are likely to be widely applicable. Advanced students often are quite articulate in their responses and explanations, thus allowing greater precision in the recognition of errors and in the identification of the underlying difficulties. Interactions with students in the classroom, particularly in tutorials, provide evidence of the extent to which they take the initiative in resolving inconsistencies in their own thinking and make connections between the physics and the increasingly sophisticated formalism to which they are introduced. Thus both formal and informal observations of students, as has been the case in similar empirical investigations in physics education research, continue to yield valuable insight into student thinking. Ongoing investigations that may involve relatively small numbers of students can promote a deeper understanding of student difficulties beyond the introductory level and guide the design and further refinement of instructional strategies that address them.

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[8]D. E. Trowbridge and L. C. McDermott, ''Investigation of student understanding of the concept of velocity in one dimension,'' Am. J. Phys. **48** (12), 1020–1028 (1980); ''Investigation of student understanding of the concept of acceleration in one dimension,'' **49** (3), 242–253 (1981).

[9]One of these additional pretests is based in part on the research presented in the articles listed in Ref. 8 and accompanies one of the tutorials included in Ref. 15.

[10]For examples of such difficulties identified by research, see R. Allain, ''Investigating the relationship between student difficulties with the concept of electric potential and the concept of rate of change,'' Ph.D. dissertation, Department of Physics, North Carolina State University, 2001; D. P. Maloney, T. L. O'Kuma, C. J. Hieggelke, and A. Van Heuvelen, ''Surveying students' conceptual knowledge of electricity and magnetism,'' Phys. Ed. Res., Am. J. Phys. Suppl. **69**, S12–S23 (2001); S. E. Kanim, ''An investigation of student difficulties in qualitative and quantitative problem solving: Examples from electric circuits and electrostatics,'' Ph.D. dissertation, Department of Physics, University of Washington, 1999 (unpublished); R. R. Harrington, ''An investigation of student understanding of electric concepts in the introductory university physics course,'' Ph.D. dissertation, Department of Physics, University of Washington, 1995 (unpublished); S. Törnkvist, K.-A. Pettersson, and G. Tranströmer, ''Confusion by representation: On students' comprehension of the electric field concept,'' Am. J. Phys. **61** (4), 335–338 (1993).

[11]See the first, second, and fourth items in Ref. 10. For instance, the first reference (Allain) details the results obtained from a diagnostic test that probes understanding of the electric field as the spatial rate of change of electric potential. The test was administered to both introductory and advanced students after instruction in electricity and magnetism. Also, in the second reference (Maloney *et al.*), item #18 on the Conceptual Survey of Electricity and Magnetism is designed to be roughly equivalent to part B of the equipotentials problem, although the contour maps given to the students on the CSEM were much simpler. Both references provide strong evidence suggesting a critical failure among students to distinguish between electric potential and electric field.

[12]L. C. McDermott, M. L. Rosenquist, and E. H. van Zee, ''Student difficulties in connecting graphs and physics: Examples from kinematics,'' Am. J. Phys. **55** (6), 503–513 (1987).

[13]Some of the examples used on the Vector curl pretest were taken from H. M. Schey, *Div, Grad, Curl, and All That: An Informal Text on Vector Calculus* (Norton, New York, 1997), 3rd ed.

[14]Among the correct explanations described here, another, more visual, approach, is described in Ref. 13, pp. 87–90. Using this approach, one can imagine a paddlewheel placed at a particular location in the $x-y$ plane such that its axis of rotation is perpendicular to this plane. If, by inspection of the force vectors in the vicinity of the paddlewheel, the net torque on the paddles is nonzero, then so must be the curl of the field at the location of the paddlewheel. The direction of the net torque is the same as the direction of the (z component of the) curl.

[15]L. C. McDermott, P. S. Shaffer, and the Physics Education Group at the University of Washington, *Tutorials in Introductory Physics* (Prentice–Hall, Upper Saddle River, NJ, 2002).

[16]For details on how tutorials have been adapted or developed for courses outside the introductory calculus-based sequence, see R. E. Scherr, P. S. Shaffer, and S. Vokos, ''The challenge of changing deeply held student beliefs about the relativity of simultaneity,'' Am. J. Phys. **70** (12), 1238–1248 (2002); B. S. Ambrose, ''Incorporating a tutorial approach in an introductory algebra-based physics course,'' AAPT Announcer **30** (2), 81 (2000); G. E. Francis, J. P. Adams, and E. J. Noonan, ''Do they stay fixed?,'' Phys. Teach. **36** (8), 488–490 (1998).

[17]L. C. McDermott, ''Oersted Medal Lecture 2001: Physics education research—the key to student learning,'' Am. J. Phys. **69** (11), 1127–1137 (2001).

[18]For research underlying these tutorials, see Refs. 5 and 8, as well as L. C. McDermott, P. S. Shaffer, and M. D. Somers, ''Research as a guide for teaching introductory mechanics: An illustration in the context of the Atwood's machine,'' Am. J. Phys. **62** (1), 46–55 (1994).

[19]The graduate students mentioned here attempted the research task on two-dimensional motion as part of a graduate teaching seminar at the University of Washington or while taking a graduate qualifying examination at the University of Washington or Montana State University. For details, see Ref. 5.

[20]For detailed discussion of difficulties that arise in the context of two-dimensional kinematics, see Ref. 5 as well as F. Reif and S. Allen, ''Cognition for interpreting scientific concepts: a study of acceleration,'' Cognition and Instruction **9** (1), 1–44 (1992).