Learning About Student Learning In Intermediate Mechanics: Using Research To Improve Instruction

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Abstract. Ongoing research in physics education has demonstrated that physics majors often do not develop a working knowledge of basic concepts in mechanics, even after standard instruction in upper-level mechanics courses. A central goal of this work has been to explore the ways in which students make—or do not make—appropriate connections between physics concepts and the more sophisticated mathematics (*e.g.*, differential equations, vector calculus) that they are expected to use. Many of the difficulties that students typically encounter suggest deeply-seated alternate conceptions, while others suggest the presence of loosely or spontaneously connected intuitions. Analysis of results from pretests (ungraded quizzes), written exams, and informal classroom observations are presented to illustrate specific examples of naïve intuitions and related difficulties exhibited by the students. Also presented are examples of instructional strategies that appear to be effective in addressing these difficulties. (Supported by NSF grants DUE-0441426 and DUE-0442388.)

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INTRODUCTION

Recent insights in physics education research (PER) have motivated the need to closely examine in new ways the nature of students' thinking before, during, and after instruction. Some conceptual and reasoning difficulties seem to be deeply entrenched, calling for instructional strategies based on cognitive conflict; other times, students seem to operate using loosely-connected or spontaneous ideas, calling instead for an approach to refine those intuitions [1]. Researchers have found it useful to attempt to characterize student thinking in introductory physics courses so that interventions in instruction can be tailored appropriately [2]. This report describes two examples highlighting the utility of extending this approach to the teaching of intermediate mechanics.

CONTEXT OF INVESTIGATION

The student populations discussed here come from junior-level mechanics courses at Grand Valley State University (GVSU), the University of Maine (UME), Seattle Pacific University (SPU), the University of New Hampshire (UNH), and West Chester University of Pennsylvania (WCUP). Although the details of the courses vary somewhat, all courses cover topics that require the synthesis of Newton's laws, work and energy, differential equations, and vector calculus. In addition, the classes highlighted here were taught by instructors implementing materials from *Intermediate Mechanics Tutorials (IMT),* a set of research-tested curricular materials modeled after published tutorials for introductory courses [3] and co-developed by both the author and Michael Wittmann (UME) [4].

The results presented in this paper were taken primarily from the analysis of responses to written pretests (ungraded quizzes), course exams, and informal observations of students during class. Unless otherwise stated, pretests were given after lecture instruction but before the relevant tutorial. All written questions asked students for explanations of reasoning.

EXAMPLE: HARMONIC OSCILLATIONS IN TWO DIMENSIONS

As reported in the 2006 PER Conference proceedings [5], research on student understanding of harmonic oscillations has revealed the often persistent intuition that the amplitude of a simple harmonic oscillator affects the period or frequency of oscillation. In the context of one-dimensional (1-D) oscillations, this intuition arose among a significant minority (~25%) of pretests given to a total of 5 intermediate mechanics classes ($N_{GVSU} = 35$ and $N_{SPU} = 11$). For example, many predicted that increasing the amplitude would cause an increase in period because, as one student explained, "the block travels farther during each period." Although most students correctly predicted no change to the period, they tended to use "compensation" reasoning [6] that amounted to plausibility arguments. As one student stated, "It may seem that the block is moving faster, but it is also moving farther to compensate." The presence of these two prominent reasoning patterns suggested the lack of a robust conceptual framework upon which students could productively build.

Inappropriate Compensation Arguments About Relative Amplitudes

The naïve student intuition linking amplitude to frequency has already been documented in the context of two-dimensional (2-D) oscillations by means of a tutorial pretest, the original version of which is discussed in Ref. 5. On this pretest students were presented several *x-y* trajectories including those shown in Fig. 1. For each case, students were asked how the frequency (or the spring constant) along the *y*-axis compared to that along the *x*-axis. (Both variations of the task yielded very similar results.)

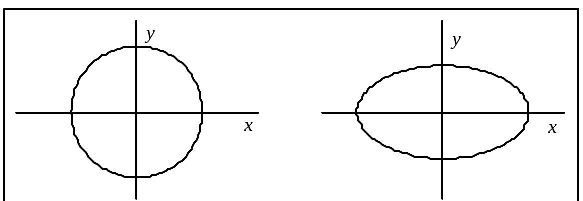


FIGURE 1. Two examples of *x-y* trajectories shown on the original pretest for 2-D oscillators. (Other examples, not shown here, included at least one non-isotropic oscillator.)

As detailed in Ref. 5, very few students answered all parts correctly. Naïve compensation arguments about the relative *x*- and *y*-amplitudes arose most prominently for Case #2—an isotropic oscillator—for which more than half of the students incorrectly answered that the frequency (or spring constant) was larger along the *y*-axis. A typical explanation would go: "The spring goes farther in the *x*-direction, so [the] spring must be less stiff in that direction."

In an effort to test whether displaying the *x-y* trajectories inordinately triggered such intuitions about the amplitudes, an alternate version of the pretest was used in two classes at GVSU ($N = 21$) starting in Fall 2007. Rather than compare frequencies or spring constants on the basis of a given *x-y* trajectory, students were instead asked to sketch a qualitatively correct *x-y* trajectory for several cases of (frictionless) isotropic and non-isotropic oscillators.

Students performed better on the isotropic cases (~40% correct) than on the non-isotropic cases (<5% correct). The amplitude-spring constant intuition accounted for many of the incorrect responses, two of which are shown in Fig. 2 for an oscillator having $k_y = 4k_x$. In the first example (Fig. 2a) the student explained that an "ellipse" would result because "the spring forces are different." Similarly, the student who drew the sketch in Fig. 2b said "the object travels less in the *y*-direction because of the stiffer spring."

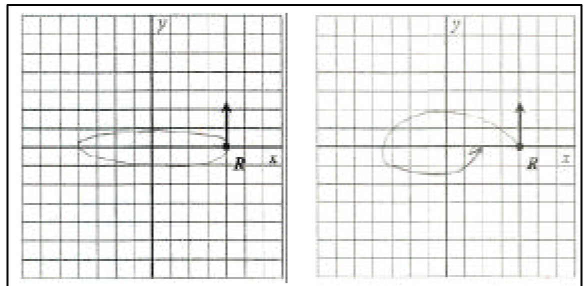


FIGURE 2. Two *x-y* trajectories drawn by students on an alternate pretest for 2-D oscillators. These sketches were drawn for a non-isotropic oscillator with $\boldsymbol{k_y = 4k_x}$.

The open-ended design of the alternate pretest elicited additional ideas not arising on the original. For instance, many students (~30%) drew inwardly spiraling paths (like the one shown in Fig. 2b) rather than closed curves, stating that "the springs attempt to return the object to equilibrium." Such a surprising result obviously warrants further research. Nonetheless, the alternate pretest seems to provide strong evidence that the amplitude-spring constant (or amplitude-frequency) intuition did not merely result from the design of the original pretest.

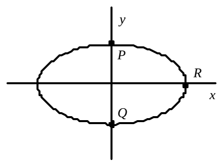
Building and Refining Productive Intuitions about Amplitude and Frequency

The in-class worksheet and homework for the tutorial "*Harmonic motion in two dimensions*" guide students to extend productively their knowledge of simple harmonic motion. Students are asked questions requiring them to extend the relationship $w = (k/m)^{1/2}$ for the frequency of an oscillator from 1-D to 2-D. A key question directs students to an *x-y* trajectory similar to Case #2 from the original pretest (Fig. 1). Students are prompted to use the diagram to find how many oscillations occur along the *y*-axis for each along the *x*-axis, a question that some students answer rather concretely by tracing their finger around the trajectory and counting. They find that, despite the different *x*- and *y*-amplitudes, the two frequencies—and hence the spring constants—are equal.

The remainder of the tutorial switches to a different issue, namely, relating the phase angle between the *x*- and *y*-oscillations for an isotropic oscillator to the initial conditions of motion. The original homework included no additional questions that sought to address the amplitude-frequency issue. After using this version of the tutorial and homework, only about half of the GVSU students in Fall 2001 and Fall 2002 (11/19 combined) correctly answered post-test tasks similar to that on the pretest. Many students instead reverted to inappropriate compensation arguments.

A. Critique the following statement. Explain your reasoning.

"The oscillator goes farther in the *x*-direction than in the *y*-direction. That means the spring in the *y*-direction must be stiffer than the spring in the *x*-direction."



B. Rank points *P*, *Q*, and *R* according to (i) total energy, (ii) potential energy, and (iii) kinetic energy.

Explain how the difference in the *x*- and *y*-amplitudes, used *incorrectly* in the statement in part A, can help you justify a *correct* answer here in part B.

FIGURE 3. Problem on the revised tutorial homework for two-dimensional oscillations.

This outcome was interpreted to mean that students continued to focus on the different *x*- and *y*-amplitudes without recognizing which aspects of the oscillations actually vary as a result. In response, a refining-intuitions approach was taken on a revised homework by including the problem shown in Fig. 3. The strategy is to help students connect the amplitude concept to a more productive one—potential energy—and thus disconnect it from spring constants and frequencies. After this modification, success rates on post-tests at GVSU have risen to ~90% (20/22 correct with correct reasoning). It is hoped that these results will soon be replicated at *IMT* pilot sites.

EXAMPLE: ANGULAR MOMENTUM AND KEPLERIAN ORBITS

After standard instruction introductory students often have difficulty applying physical relationships (*e.g.*, ideal gas law) that involve multiple variables [7]. These difficulties seem to be particularly prevalent for cause-effect relationships (*e.g.*, work-energy and impulse-momentum theorems) [6]. Because difficulties frequently resurface after instruction at the introductory level, these findings suggest possible confusion about angular momentum of object moving under the influence of central forces.

Spontaneous Intuitions About Angular Momentum

A pretest was administered to several classes with the goal of probing student thinking about the angular momentum of a point particle. Two scenarios that elicited the most interesting results are shown in Fig. 4 below. For the particle moving with uniform motion (Fig. 4a), students were asked whether or not its angular momentum changed. For the comet (Fig. 4b), students were asked to rank points *A*–*D* along the orbit according to the angular momentum of the comet.

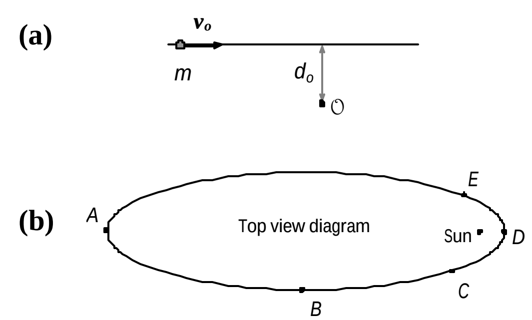


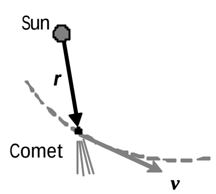
FIGURE 4. Diagrams from the tutorial pretest on Kepler's second law. **(a)** A point particle moving with uniform motion. **(b)** Diagram of the orbit of a comet. (Students were told explicitly ***not*** to treat the picture as a strobe diagram.)

Students who took the pretest after lecture (N_{WCUP} = 22 and N_{UNH} = 7) found them to be quite difficult. Only 3/29 (~10%) gave correct answers on the comet question supported by correct and complete reasoning (*i.e.*, torque by gravitational force is zero). Another ~60% (18/29) gave correct responses but, like those on the 1-D oscillator pretests, many of these were based on compensation reasoning (*e.g.*, "all equal since if *v* is increasing, *r* is decreasing, and vice versa") instead of cause-effect arguments. In addition, some students correctly stated that the comet's angular momentum would not change "because gravity is a conservative force," as if to assert that any force that conserves energy also conserves angular momentum.

Of even more interest were the intuitions used by students giving incorrect answers to either pretest task. For example, most students incorrectly stated that the particle in Fig. 4a would have changing angular momentum, referring to *only* the distance from the reference point ("the distance to O changes while the speed stays the same") *or* the angle between position and velocity ("the angle that the radius makes with the [path] is constantly changing"). Similarly, some students based their angular momentum ranking for the comet (Fig. 4b) only on *one* variable, *e.g.*, speed: "as the object moves closer to the sun, *v* increases, so momentum increases." Still others focused on the radius of curvature, suggesting a link to angular velocity: "*A, D > C > B… A* & *D* must have the most angular momentum to execute the tightest turn."

Guiding Students To Deduce New Physics Through Guided Derivations

The frequent use of compensation arguments indicate a link between angular momentum and (at least some of) the constituent quantities in its definition. Unfortunately, the relationship between torque and angular momentum—as with many dynamical cause-effect relationships—escapes many students. The tutorial "*Angular momentum and Kepler's second law*" is designed to guide students through the reasoning in order to arrive at this relationship and recognize how to apply it.



$$\frac{d\vec{L}}{dt} = \frac{d}{dt}\left(\vec{r} \times m\vec{v}\right)$$
$$= \left(\frac{d\vec{r}}{dt} \times m\vec{v}\right) + \left(\vec{r} \times m\frac{d\vec{v}}{dt}\right)$$

FIGURE 5. Diagram and initial steps of a guided derivation given to students in the tutorial on Kepler's second law.

The heart of the guided derivation rests in asking students to explain the need for and meanings of both terms from the time derivative of angular momentum (see Fig. 5). Students generally recognize that the first term is identically zero, although some need explicit guidance to do so. (Some initially interpret "$d\vec{r}/dt$" to mean the rate of change of the *magnitude* of position.) Students must also interpret the second term as the torque produced by the net force. In this way students deduce that the comet experiences zero net torque, and hence its angular momentum is conserved.

Post-test performance on exams suggests modest improvement. As a follow-up to the comet question on the pretest, post-tests were given to 2 classes at GVSU and 1 class at SPU. One GVSU class (N = 15) was asked whether or not an attractive $1/r^3$ force (posited as a "hypothetical gravitational" force) would conserve momentum; 60% (9/15) correctly explained in the affirmative noting that force and position would be antiparallel, causing zero torque. The other classes (N_{GVSU} = 8, N_{SPU} = 11) were asked the same question for a force expressed in Cartesian coordinates. Only ~40% (8/19) answered correctly. The lower success rate is likely due to the need for students to explicitly calculate the torque by the force. On both post-tests the most common error (8/34) was based on the notion—a deeply-entrenched idea?—that conservative forces must also conserve angular momentum.

CONCLUDING REMARKS

This report has sought to provide examples by which reformed instruction in intermediate mechanics has enhanced student learning by attending to the coherence and stability (or lack thereof) in the thinking and reasoning patterns of the students. The refinement of student intuitions and the careful use of guided derivations appear to be promising instructional strategies for upper division physics students.

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REFERENCES

1. A. Elby, *Am. J. Phys. Suppl.* **69**, S54-S64 (2001).
2. R. E. Scherr, *Am. J. Phys.* **75**, 272-280 (2007).
3. L. C. McDermott, P. S. Shaffer, and the Phys. Ed. Group at the U. of Washington, *Tutorials in Introductory Physics*, Prentice-Hall: Upper Saddle River, NJ, 2002; M. C. Wittmann, R. N. Steinberg, E. F. Redish, and the Phys. Ed. Res. Group at the U. of Maryland, *Activity-Based Tutorials*, Wiley: New York, 2004.
4. B. S. Ambrose, *Am. J. Phys.* **72**, 453-459 (2004).
5. B. S. Ambrose, "Probing Student Reasoning and Intuitions in Intermediate Mechanics: An Example with Linear Oscillations" in *Physics Education Research Conference-2006*, edited by L. McCullough et al., AIP Conference Proceedings 883, American Institute of Physics, Melville, NY, 2007, pp. 30-33.
6. T. O'Brien Pride, S. Vokos, and L. C. McDermott, *Am. J. Phys.* **66**, 147-156 (1998).
7. C. H. Kautz, P. R. L. Heron, M. E. Loverude, and L. C. McDermott, *Am. J. Phys.* **73**, 1055-1063 (2005).